Back numbers

This is the first issue of the Newsletter Articles Supplement with the new name **Archetype**. *The contents of the issues 1, 2 and 3 are listed below*.

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Elements of a differential and integral calculus in counterspace

P. P. Veugelers

1. Introduction

In my paper 'Thermal expansion in counterspace' (Newsletter Articles Supplement 3, 1997), I developed a procedure for determining the magnitude of a surface area in counterspace.[1] The use of this procedure however, is limited to quadrangles of which the two opposite sides are perpendicular to each other. To be able to determine the magnitude of a surface area with an arbitrary other shape, I have extended this procedure to a procedure for integration in counterspace. The next logical step was to also develop a procedure for differentiating in counterspace, to lay the foundations for an integral and differential calculus in counterspace. With this I hope to contribute to a development of mathematics that will make it possible to describe physical phenomena with a 'counterspatial signature' scientifically and quantitatively.

I consider it important that mathematical treatments can be connected with representations. In my opinion, abstraction is only called for in a following stage. Therefore my approach to the subject is such that the reader (and myself) have as much hold as possible on (geometric) representations.

In this paper, I will restrict myself to two-dimensional space. In section 2 the basic elements of Euclidean and counterspace are treated, namely the point and the line, as well as the determination of the position of these elements, join and intersection.

In section 3 I develop how a function in Euclidean space can be described as a function in counterspace.

In section 4 I develop a procedure for the determination of distance in counterspace.

In section 5 I develop a procedure for the determination of surface area in counterspace.

Section 6 treats the differentiation of a function in counterspace.

Section 7 treats the integration of a function in counterspace, with which also the surface area of an arbitrary shape in counterspace can be determined.

In the conclusion, section 8, the most important results are summarised.

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1. With my research on the principles of an integral calculus in counterspace it appeared that the mathematics of counterspace complements those of Euclidean space, only if with the transformation of one space to the other, use is being made of polar-reciprocal inversion (reciprocation) with respect to an *imaginary* unit circle. In my paper 'Thermal expansion in counterspace' I used the common polar-reciprocal inversion with respect to a *real* unit circle. However, the contents and the conclusion of this paper are not affected by this, because the use of an imaginary unit circle only results in an inversion of the signs of the co-ordinate axes in counterspace. This inversion of signs has no consequences for the determination of surface area in counterspace because this is calculated with the absolutes of the relevant distances.

2. Determination of position in space and counterspace

2.1 Point co-ordinates

In projective geometry, the position of a point on a line is unambiguously determined by three reference points on this line and a number which is called the cross ratio. If the three

Figure 2.1

reference points are called O_1, U and O_2, respectively, and the point to be determined is called X (figure 2.1), then the cross ratio is defined as:
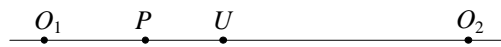
$$r \equiv (O_1 P U O_2) \equiv \frac{O_1 P}{O_1 U} \div \frac{P O_2}{U O_2} \qquad (2.1)$$

Here $O_1 X$ is the length of the line segment between O_1 and X if the line is traced from O_1 via X to O_2. $X O_2$ is the length of the line segment between X and O_2 in the same direction. $O_1 U$ and $U O_2$ are defined similarly.

If O_1 is placed at the origin of Euclidean space (O), O_2 at the infinity of Euclidean space (∞) and $O_1 U$ is defined as the unit length, then the cross ratio passes into the Euclidean co-ordinate x. For then $P O_2 = U O_2 = \infty$ and $O_1 U = 1$ and therefore:

$$r = (O P U \infty) = O P = x_P \qquad (2.2)$$

In two dimensional projective space, the position of a point is unambiguously determined by four reference points and two cross ratios, as shown in figure 2.2. Here O_1, O_2, O_3 and U are the reference points and P the point to be determined. The cross ratios are respectively:

$$r_1 \equiv (O_3 P' U' O_1) \equiv \frac{O_3 P'}{O_3 U'} \div \frac{P' O_1}{U' O_1} \qquad (2.3)$$

and

$$r_2 \equiv (O_3 P'' U'' O_2) \equiv \frac{O_3 P''}{O_3 U''} \div \frac{P'' O_2}{U'' O_2} \qquad (2.4)$$

where P' and U' are the projections of respectively P and U on o_2, and P'' and U'' are the projections of respectively P and U on o_1.

Now place O_3 at O, the origin of Euclidean space, O_1 and O_2 in the infinite of Euclidean space, o_1 and o_2 perpendicular to each other and $O_3 U'$ and $O_3 U''$ equal to the unit length. Then both cross ratios pass into the Euclidean co-ordinates x and y of P. For then $P' O_1 = U' O_1 = \infty$ and $O_3 U' = 1$ and therefore:

$$r_1 = (O P' U' \infty) = O P' = x_P \qquad (2.5)$$

and $P'' O_2 = U'' O_2 = \infty$ and $O_3 U'' = 1$ and therefore:

$$r_2 = (O P'' U'' \infty) = O P'' = y_P \qquad (2.6)$$

assumptions are secondary assumptions that are always evaluated in every rigorous geochronological study. Experience has led to numerous criteria through which the validity of these assumptions can be tested, and thus the distinction made between meaningful and spurious ages. It is misleading, therefore, to present this single fundamental assumption as if it is typical of all assumptions, because it is not. You will find a limited attempt in my radioactivity paper to evaluate the single fundamental assumption., Third, ^{14}C disintegrates to ^{14}N, not ^{12}C as you state. Finally, if you wish to propose that decay constants have varied in the past and the Earth is younger than conventionally thought, it is surely incumbent on you to also propose how might this be detected in the Earth's history. It seems to me that if no criteria can be established to evaluate whether the constants have been constant or have varied, then the idea that they might have varied is not a scientific hypothesis but a belief whose substance lies only in its meaningfulness to the individual believer.

December 5, 1997

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1. This letter responds also to Ron Jarman's contribution at the 'Age of the Earth' conference of the Science Group on 22-23 November 1997 at Wynstones School, Gloucester. The notes of Ron Jarman's contribution were included in the March 1998 issue of the Science Group Newsletter (pp 4-6) and further details are given in Derek Forman's report on the conference in the same issue (pp 3-4).

geochronological years correspond to calendar years, the evidence that radioactive decay has been a property of the Earth for all of its material history, and the conclusion that radioactive decay is a memory of the Old Saturn stage in Earth evolution. The question of the geochronological age of the Earth raises quite other issues, and has to be argued in technical ways that incorporate measurements made on meteorites as well as terrestrial materials.

In the other I make the point that in the early years of Steiner's leadership of the German section of the Theosophical Society the influence of Theosophy on Steiner's early work ran much deeper than many Anthroposophists are aware or even interested in. A number of people have reacted in ways similar to you, that I damn Steiner by faint praise, one called me a liar. I take the position that Steiner lived intensely in his life and times, and so we should expect to find their influence in his work - particularly, as you point out, in the early years of his leadership in the German Theosophical Society - and I do not accept that this diminishes Steiner's originality. The vitality of the many enterprises that have come from Anthroposophy testify to this originality. I was struck this year when visiting the Theosophical headquarters how dead and museum-like it is, in complete contrast to the life and vitality in the Goetheanum.

There seem to me two extreme positions to take on Steiner's work, neither of which is sustainable. One is that everything in Anthroposophy is original to Steiner's spiritual research. This was the position I took for most of my life, until I was profoundly shocked to discover this is not the case when I casually opened a book on Theosophy. The other position is to say that yes, there are Theosophical influences in Steiner's early work after 1902, but they are there only because he was able to independently verify them. I believe this to be a very hazardous position to take, particularly for Anthroposophists who are not familiar with the fundamental Theosophical books, the full history of the Theosophical Society and biographies of the leading Theosophists. To uncritically link Anthroposophy to Theosophy goes against the efforts Steiner made late in life to separate the two, and burdens Anthroposophy with the human frailties and foolishnesses that are so evident in the history of the Theosophical Society. Surely, Anthroposophy has enough opposition without having to defend Theosophy?

I cannot comment on Ron Jarman's notes because they represent a flow of thoughts that I, of my own initiative, would not think of connecting together. The exception is section D, for which I offer the following. First, I think most earth scientists would feel surprise at the assertion that geochronology is "the most popular (and vociferous) school of geological thought today". Second, I think that many readers might be misled by the reference to many assumption in geochronology, because the "many assumptions" argument is the usual argument Anthroposophists use to uncritically dismiss radioactive dating. There is one fundamental assumption, that the number of disintegrations per unit time is proportional to the quantity or number of atoms of the radioactive parent. This is the single fundamental assumption that is never questioned, although there were attempts to evaluate it in the 1930s.. All the other
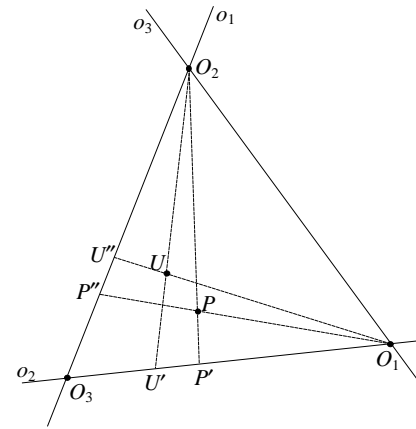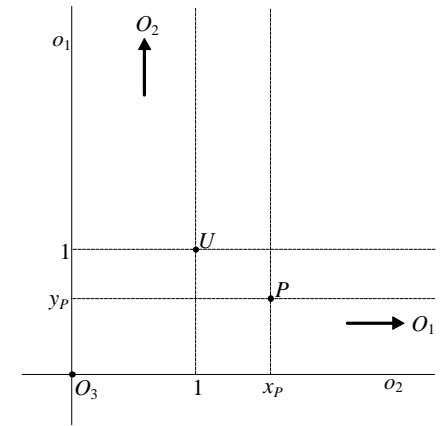
This situation is shown in figure 2.3.

Figure 2.2

Figure 2.3

The pair (x, y) is called the *point co-ordinates* of a point in Euclidean space.

To avoid the number ∞, use is also being made of so called '*homogeneous point co-ordinates*'. These are noted (x_1, x_2, x_3) with $x = \dfrac{x_1}{x_3}$ and $y = \dfrac{x_2}{x_3}$. From this it follows directly that the point (x_1, x_2, x_3) is identical to the point (lx_1, lx_2, lx_3) with l an arbitrary constant.

A point in the infinite has homogeneous point co-ordinates $(x_1, x_2, 0)$, where the ratio of x_1 to x_2 determines the direction in which this point lies.

If O_1 and O_2 are placed in the origin of *counterspace* (i.e. the infinite of Euclidean space), O_3 in the infinite of *counterspace* (so in O), o_1 and o_2 again perpendicular to each other and $U'O_1$ and $U''O_2$ equal to the unit length in *counterspace*, then $O_3P' = O_3U' = -\infty$ and $O_1U' = -1$ and the cross ratio on the line o_2 passes into the co-ordinate in counterspace which I shall call u:

$$r_1 = (\infty P'U'O) = -\frac{1}{OP'} = u_P \qquad (2.7)$$

Further $O_3P'' = O_3U'' = -\infty$ and $O_2U'' = -1$ the cross ratio on the line o_1 passes into the co-ordinate in counterspace which I shall call v:

$$r_2 = (\infty P''U''O) = -\frac{1}{OP''} = v_P \qquad (2.8)$$

The pair (u, v) is called the point co-ordinates of a point in counterspace. In homogeneous point co-ordinates this becomes (u_1, u_2, u_3) with $u = \dfrac{u_1}{u_3}$ and $v = \dfrac{u_2}{u_3}$.

Figure 2.4 shows a co-ordinate system with both Euclidean and counterspatial co-ordinates. The x co-ordinate is plotted at the top of the horizontal axis, the u co-ordinate at the bottom. The y co-ordinate is plotted at the right side of the vertical axis, the v co-ordinate at the left side.
From equations (2.5) and (2.7) respectively (2.6) and (2.8) it follows that:

$$u = -\frac{1}{x}, \quad v = -\frac{1}{y} \qquad \textbf{(2.9)}$$

which also follows from figure 2.4. Equation (2.9) may be written in homogeneous co-ordinates as:

Figure 2.4

$$u_1 = -\frac{1}{x_1}, \quad u_2 = -\frac{1}{x_2}, \quad u_3 = \frac{1}{x_3} \qquad \textbf{(2.10)}$$

2.2 Line co-ordinates
The natural basic element in Euclidean space is the point. This arises from the fact that the zero element of Euclidean space is a point. Every other point is in fact a copy of this zero element, translated over a distance in the x direction and a distance in the y direction according to the co-ordinates of the point. To be sure, the infinite element of Euclidean space is a line, but this line cannot be affected by translations (or rotations) and therefore is not suitable as a basic element. In fact, a line is a 'stranger' in two dimensional Euclidean space. This is for instance expressed in the fact that a line is represented by a set of *points* (x, y) that satisfies the equation:

$$y = ax + b \qquad \textbf{(2.11)}$$

with a and b constants.
The line in the infinite of Euclidean space, however, is the zero element in the two dimensional counterspace. Because of this, the line is the natural basic element of counterspace.
The position of a line is determined by the abscissas in counterspace. For instance, a line a: $[u_a, v_a]$ has u abscissa u_a and v abscissa v_a (figure 2.5). The pair $[u, v]$ is
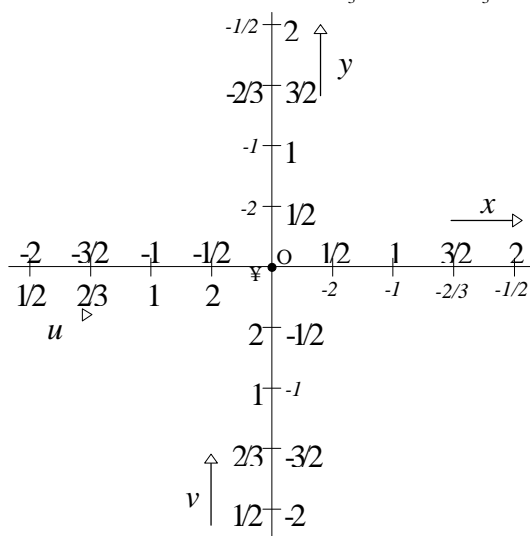
with cosmology consistent with our knowledge of the Earth at the end of the 20th century distinct from the mere theories of a particular school of geology. He would also have developed his Science of the Spirit much further, for example paying much more attention to the Third Force, i.e. of the Asuras whose aim is the illness and death of human culture. No longer can we just see ourselves treading the tightrope between Lucifer and Ahriman. There is a trinity of temptations today (p 38). Lemuria is certainly neither a fixed nor literal object, but is a reality amenable only to spiritual research. To focus its centre in the Indian Ocean has a lot of sense to people who have made themselves aquainted with Aboriginal culture. (I was in Australia for 2 months this summer).

One of the most difficult and profound lectures given by Steiner in his later years was on December 1st 1923 – lecture 5 of *Mystery Centres*. There he described the Lemurian epoch. To quote one sentence, "As regards those very ancient times it is really not possible to speak of oxygen, nitrogen, carbon, sulphur and so on, simply because what the chemist calls by those names did not exist in that ancient period." Again had you queried Steiner at the time, he would have replied, "Don't take my word for it, develop the necessary organs of perception and see for yourself."

25th November 1997

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Response to Ron Jarman,[1]

My main reaction to Ron Jarman's comments is to be reminded that everything we write is in some sense autobiographical. I, therefore, see no reason to try to assert that either he or I are right or wrong on certain issues, because this runs the danger of denying a person's life experience. Steiner's influence on my understanding of many things runs deep in me, with the exception of everything he wrote about the Earth and its history. Steiner's descriptions of the Earth can be found in the scientific, popular and esoteric movements on the late 19th century, none are prescient with respect to the late 20th century, and some are clearly implausible. In stating this, I take Steiner literally, as when he describes Atlantis as lying on the floor of the Atlantic between Africa and North America, or when he asserts that the cause of the recent ice age was the elevation of continents.

I do think that Ron Jarman wrote about issues that I did not raise, namely the "age" of the Earth on the one hand, and the influence of Theosophy on <u>all</u> of Steiner's work. In the article on radioactivity I discussed the intimate connection between life and radioactive decay, the idea that radioactivity has sustained the heat body of the Earth, the evidence and the time spans over which we can be confident that

Correspondence

The following two letters relate to Norman Grant's articles in the 1996 issue of the Newsletter Articles Supplement (No.2) entitled 'Radioactivity in the history of the Earth' (pp 12-30) and 'Steiner's description of the Earth's history' (pp 31-39).

To Norman Grant:

Whilst recognising your clear exposition of the modern popular theory of geochronology, I have to take issue with your statements and views about Rudolf Steiner's pronouncements. Laudable though it may at first sight seem to excuse this undoubtedly wise man because of his misfortune to have lived in the first quarter of our century instead of the last, the general effect of your analysis is to damn him with faint praise. But is your analysis correct ?

The fact that Steiner, Blavatsky and others were initially "tutored" by the same Master does not permit one to deduce that Steiner's knowledge and wisdom developed solely from that source. Steiner's initiate knowledge had far deeper spiritual sources, the consequence of his work both in previous earth lives and in this one. Naturally he had to link on to Theosophical and other spiritual strivings current during his earlier years in order to be heard at all.

It just won't do to suggest that (Steiner) ⌐ (Blavatsky), i.e. that Steiner and his work is a subset of Blavatsky and hers. Moreover if you examine Steiner's prefaces in the later editions of his *Knowledge of the Higher Worlds* it is clear that it is no longer necessary to find a personal Master. The essential thing about Anthroposophy is that it is a scientific path of initiation and that Steiner is a very reliable guide, a friend not a guru. What he has to say in this or that lecture (and many of the stenographed reports are not wholly reliable – it was Steiner's sorrow that he hadn't the time to correct them) should only be taken as indications which in course of time the reader must verify, modify or reject. This requires initiation and it should not be assumed that anthroposophists have not achieved this. Several of us know quite clearly who we were in earlier incarnations, for example. When I lived in ancient Troy, the carboniferous limestone rocks were both a bit softer and had a different colour (light blue), but I can't give a brain-bound proof of that. The sea, too, had a different; colour.

A few comments about the text of your second article "Steiner's description of the Earth's History" (pp 33-34) Steiner consulted *The Secret Doctrine* in order to confirm his own research (p 34). There were certainly issues that brought Theosophy into disrepute, particularly Annie Besant's insistence on the reincarnation of Christ as Krishnamurti. No wonder that Steiner distanced himself from the Theosophists (pp 35-36). Once again Steiner's reference to "theosophical books" does not mean that they were his authority for his descriptions of Lemuria (p 37). Anthroposophy functioning as a museum – yes, that is just how materialists would like to regard it (p 37). There is a third answer to your question – that Steiner would have responded

called line co-ordinates (note the *square* brackets). In particular the origin of counterspace (i.e. the line in the infinite of Euclidean space) has line co-ordinates [0, 0].

Analogous to how in section 2.1 homogeneous point co-ordinates were introduced, now also homogeneous line co-ordinates may be introduced. These are written $[u_1, u_2, u_3]$ with $u = \dfrac{u_1}{u_3}$ and $v = \dfrac{u_2}{u_3}$.

A point now may be represented as a set of lines $[u, v]$ which satisfies the equation:

$$v = cu + d \qquad (2.12)$$

with c and d constants.

With $x = \dfrac{c}{d}$ and $y = -\dfrac{1}{d}$ this equation may also be written as:

$$xu + yv + 1 = 0 \qquad (2.13)$$

with x and y constants.
The last equation, however, may also be used to determine a line as a set of points. This follows from the substitution of $u = \dfrac{a}{b}$ and $v = -\dfrac{1}{b}$ in (2.11) which also results in (2.13), but now with u and v constants and x and y variables. Thus equation (2.13) stands on the one hand for all lines $[u_i, v_i]$ through the point (x, y) and on the other hand for all points (x_i, y_i) on the line $[u, v]$. Figure 2.6 shows both concepts. Equation (2.13) in homogeneous co-ordinates is:

Figure 2.6

$$x_1 u_1 + x_2 u_2 + x_3 u_3 = 0 \qquad (2.14)$$

With x_1, x_2 and x_3 constants this is then the equation of a set of lines through a point (x_1, x_2, x_3) and with u_1, u_2 and u_3 constants the equation of a set of points on a line $[u_1, u_2, u_3]$.

2.3 Incidence, intersection and join
Equation (2.13) represents on the one hand the set of all lines through the point (x, y) and on the other hand the set of all points on the line $[u, v]$. Therefore, when a point and a line both satisfy (2.13), the point must lie on the line. This is called 'incidence'. The equation:
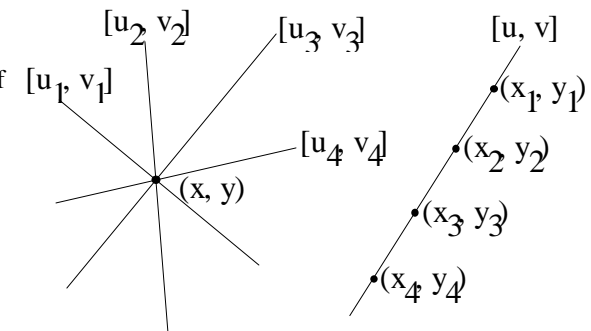
$$xu + yv + 1 = 0 \qquad \textbf{(2.15)}$$

therefore is called the 'condition of incidence' of a point and a line. In homogeneous co-ordinates this equation is:

$$x_1 u_1 + x_2 u_2 + x_3 u_3 = 0 \qquad \textbf{(2.16)}$$

For the point of intersection (x, y) of two lines $[u_a, v_a]$ and $[u_b, v_b]$ we have:

$$
\begin{aligned}
xu_a + yv_a + 1 &= 0 \\
xu_b + yv_b + 1 &= 0
\end{aligned}
\qquad \textbf{(2.17)}
$$

Solving these equations gives:

$$x = \frac{v_a - v_b}{u_a v_b - u_b v_a}$$

$$\qquad \textbf{(2.18)}$$

$$y = \frac{u_b - u_a}{u_a v_b - u_b v_a}$$

In homogeneous co-ordinates we have for the point of intersection (x_1, x_2, x_3) of two lines $[u_1, u_2, u_3]$ and $[v_1, v_2, v_3]$:

$$
\begin{aligned}
x_1 u_1 + x_2 u_2 + x_3 u_3 &= 0 \\
x_1 v_1 + x_2 v_2 + x_3 v_3 &= 0
\end{aligned}
\qquad \textbf{(2.19)}
$$

Solving these equations gives:

$$
\begin{aligned}
x_1 &= \lambda\,(u_2 v_3 - u_3 v_2) \\
x_2 &= \lambda\,(u_3 v_1 - u_1 v_3) \\
x_3 &= \lambda\,(u_1 v_2 - u_2 v_1)
\end{aligned}
\qquad \textbf{(2.20)}
$$

where λ is an arbitrary constant.

For the join $[u, v]$ of two points (x_A, y_A) and (x_B, y_B) we have:

$$
\begin{aligned}
x_A u + y_A v + 1 &= 0 \\
x_B u + y_B v + 1 &= 0
\end{aligned}
\qquad \textbf{(2.21)}
$$

Solving these equations gives:

$$u = \frac{y_A - y_B}{x_A y_B - x_B y_A}$$

$$\qquad \textbf{(2.22)}$$

$$v = \frac{x_B - x_A}{x_A y_B - x_B y_A}$$

4. E. Guyenot, Les sciences de la vie aux XVIIe et XVIIIe siecles, Paris, 1941, p. 173 (L'evolution de l'humanite, no. 68). Quoted in J. H. van den Berg, Het Menselijk Lichaam, 1962.

5. J. H. van den Berg, Foreword to Psychological Life: From Science to Metaphor, by Robert D. Romanyshyn (Austin, University of Texas Press, 1982), pp. xi, xii.

6. Owen Barfield, Saving the Appearances: A Study in Idolatry (New York: Harcourt Brace Jovanovich, 1965), pp. 49-51. See also Pierre Duhem, To Save the Phenomena: An Essay on the Idea of Physical Theory from Plato to Galileo (Chicago: University of Chicago Press, 1969), chapter 7.

7. Charles Siebert, "The Rehumanization of the Heart," Harpers, February, 1980, pp. 53-60.

8. What follows is a summary drawn from portions of three or four lectures delivered by Incao between 1989 and 1995.

9. Herbert Butterfield, The Origins of Modern Science (New York: Free Press, 1957), pp. 13-14.

10. Owen Barfield, "Thinking and Thought," reprinted in Romanticism Comes of Age.

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This essay was intended as an introduction to a longer work dealing with the nature of the heart. The longer work is currently gestating.

deadening distance between himself and the life that might otherwise break in upon him? One suspects that an obstetrician who will not hear of the womb can, in a very real sense, deliver only stillborn children.

But we are all the offspring of our age, and in one way or another we employ the same disinfectant when crossing a certain internal threshold. All of us have experienced, for example, the aching constriction of heart following a personal loss. But the aching heart, as also the light and heavy heart, the courageous heart, the wise heart, and the loving heart – these, if we even notice their murmurings, become mere background noise the moment we present ourselves to the cardiologist for an examination. We can scarcely imagine that the ignored voices might be relevant to the jagged spikes of the electrocardiogram. The threshold that Harvey crossed lives on as a split within each of us – except that the one side of the threshold has faded into little more than an "ancient memory" for which we can no longer find meaning or justification.

The split is even visible within science itself. As Barfield observed several decades ago, "in Medicine, the whole of the surgical branch has reached a point little short of perfection; but when it is a question of treating malignant growths and, in general, diseases of the living organism, where are we?"[10]

The fact is that disease of the heart is more rampant in our technically enlightened society than ever before in history. We thought that we knew enough to create and implant artificial hearts; the reality is that we haven't even had the wisdom to prevent our natural hearts from deteriorating at epidemic rates.

Are our modern hearts able to bear any longer the burden of being treated like mechanisms? The question is a valid one if the contemporary urge toward holism is at all justified – if the health of the human organism depends upon our healing the split between consciousness and matter, between our innermost experience and the data of science, between the "whispering, wailing, loving, longing tale" at the heart of our being and the machine that Harvey first heard beating.

Every orthodoxy, every received wisdom, has its one-sidedness and limitations, which must be transcended in due time lest decay set in. A science whose fundamental character was determined at birth by the fact that machines were then gaining prominence is, after all, likely to be an imbalanced science, and the imbalances are likely to become ever more acute as the science matures. How better to seek the terms of a renewed wisdom than by listening afresh to the throbbing Balance and Center of the human being?

Notes

1. Robert D. Romanyshyn, Psychological Life: From Science to Metaphor, p. 74
2. G. Joseph, G, Geschichte der Physiologie der Herztoene vor und nach Laennec bis 1852, in Janus, 1931. Quoted in J. H. van den Berg, Het Menselijk Lichaam, 1962.
3. Emilio Parisano, Recentiorum disceptationes de motu cordis, sanguinis et chyli (Leiden, 1647), p. 107. Quoted in J. H. van den Berg, Het Menselijk Lichaam, 1962.

In homogeneous co-ordinates we have for the join $[u_1, u_2, u_3]$ of two points (x_1, x_2, x_3) and (y_1, y_2, y_3):

$$x_1 u_1 + x_2 u_2 + x_3 u_3 = 0$$
$$y_1 u_1 + y_2 u_2 + y_3 u_3 = 0 \qquad \textbf{(2.23)}$$

Solving these equations gives:

$$u_1 = \lambda(x_2 y_3 - x_3 y_2)$$
$$u_2 = \lambda(x_3 y_1 - x_1 y_3) \qquad \textbf{(2.24)}$$
$$u_3 = \lambda(x_1 y_2 - x_2 y_1)$$

where λ is an arbitrary constant.

3. Polar-reciprocal inversion w.r.t. an imaginary unit circle

3.1 Polar-reciprocal inversion of a point and a line

Figure 3.1 shows the construction of the polar-reciprocal inverse of a point A w.r.t. an imaginary unit circle[2]. The image of A is the line a. The figure is self explanatory. When the point A lies not as in figure 3.1 outside the unit circle, one chooses through A two arbitrary lines l and m and constructs from these the inverse points L and M according to figure 3.1. The join of L and M is the inverse a of A.
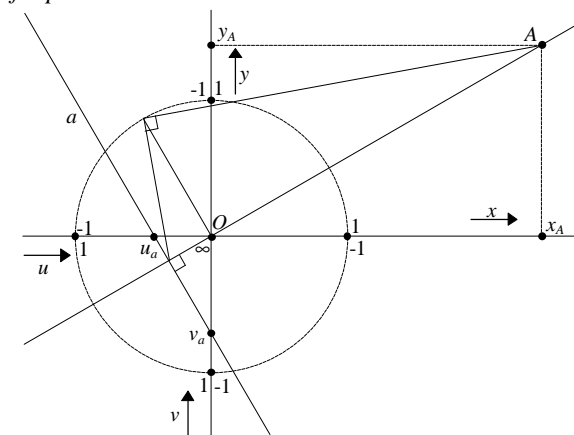
With help of some elementary geometry it is simple to demonstrate that $u_a = x_A$ and $v_a = y_a$. From this it follows that for the connection between the point co-ordinates of an arbitrary point (x, y) and the line co-ordinates $[u, v]$ of the polar-reciprocal image of it:

Figure 3.1

$$(x, y) \xrightarrow{\substack{\text{polar} \\ \text{reciprocal} \\ \text{inversion}}} [x, y]$$
$$\qquad \textbf{(3.1)}$$
$$[u, v] \xrightarrow{\substack{\text{polar} \\ \text{reciprocal} \\ \text{inversion}}} (u, v)$$

---

[2] For the theory of this see part 22 of: Lawrence Edwards, Projective Geometry, 1985, Rudolf Steiner Institute.

or in homogeneous co-ordinates:

$$(x_1, x_2, x_3) \xrightarrow{\substack{polar \\ reciprocal \\ inversion}} [x_1, x_2, x_3]$$

$$[u_1, u_2, u_3] \xrightarrow{\substack{polar \\ reciprocal \\ inversion}} (u_1, u_2, u_3)$$

(3.2)

3.2 Polar-reciprocal inversion of a function

By polar-reciprocal inversion w.r.t. an imaginary unit circle, every point of a curve in the Euclidean plane is transformed into a line. These lines form the envelope of a curve in counterspace. In figure 3.2 from a function $y = f(x)$ sixteen points are marked. These points transform in sixteen tangents to the inverse function.

Of the original function which consists of all points $(x, y) \mid y = f(x)$, every point $(x, f(x))$ is transformed into a line $[u, v]$, for which according to (3.1) we have:

$$(x, f(x)) \xrightarrow{\substack{polar \\ reciprocal \\ inversion}} [x, f(x)]$$

(3.3)

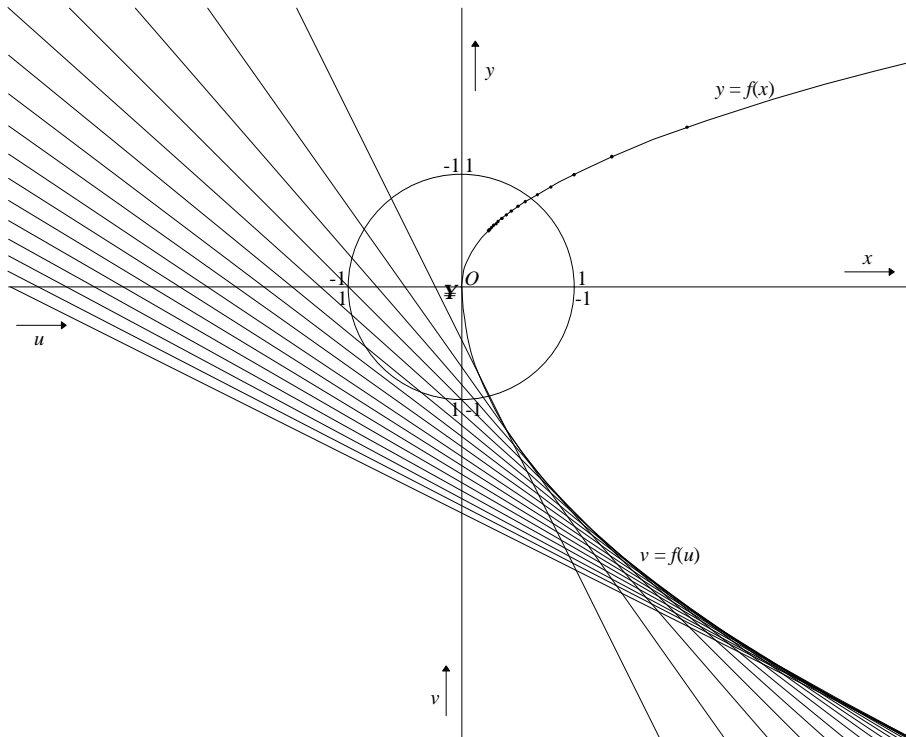Therefore, the original function is transformed into a function in counterspace which is

Figure 3.2

the venous blood – artificial respiration was begun. Within ten minutes the oxygen content of the arterial blood rose to normal levels, indicating a persistent, if slowed, blood circulation in the absence of a beating heart.

Questions

We are justified, it seems in stepping back and trying to piece together a fuller, more organic context for the beating heart.

When imported into the life sciences, mechanical notions – whether of a pump or any other device – tempt us to discard part of what we know about the living organism. We naturally desire to achieve the same clear chains of cause and effect that seem so serviceable when applied to automobiles and washing machines. But we can achieve that one-dimensional clarity only by sacrificing the rich interplay through which the whole organism lives in every part and every part serves the whole. After all, when the wheel learns to fiddle with the carburettor's adjustment, things begin to get complicated – perhaps, in the living organism, even complicated enough to make addressing the recalcitrant heart through an incision in the arm a reasonable gesture.

There are no isolated systems in a living organism. Every aspect of its being expresses every other aspect. Finding an appropriate and disciplined language for this organic relatedness is the challenge confronting any holistic science. Unfortunately, most alternatives to mechanistic explanation today present us with a choice between obscurantism on the one hand, and what turns out to be a mere elaboration of mechanism (as in feedback mechanisms and computational mechanisms) on the other hand. This is hardly surprising, given our centuries-long training in quantitative and mechanistic styles of thought.

Where Harvey lived between two eras and passed from one to the other by watching the heart as it died, we, the children of the scientific era, have buried the threshold he crossed deep within ourselves. It lives on now only as dimly reflected in the unequal contest between poetic and scientific truth, between metaphor and fact, between the vagaries of qualitative experience and the certainties of reliable, quantitative data. Harvey killed off his living specimens to gain a knowledge of the heart's mechanism; we compulsively vivisect, so to speak, our own living hearts in favour of inert, mechanical ones.

I was reminded of this when my wife, Phyllis, told me about her opportunity to observe a caesarean birth while in nursing school. After the delivery, the surgeon pointed to one of the mother's exposed abdominal organs and asked the students, "What is this?" Without thinking, Phyllis immediately responded, "the womb." It was the wrong answer. Visibly irritated, the surgeon corrected her with caustic emphasis: "The proper term is `uterus'."

It was as if this physician, charged with attending at the threshold of a fellow human being's emergence into the light of this world, could not tolerate any but the most detached terminology for the miracle he assisted. Did he feel, somewhere inside, that his entire training depended for its validity upon his ability to put a

must show a certain deference toward authority. On the other hand, the effort to see things in a shifted light, to look for a different sort of possibility in the appearances lying before one's eyes, is a task to which the greenest student (an apt metaphor in the dissecting laboratory!) might usefully contribute – in fact, might especially contribute, due to his very inexperience – if only he has previously learned how.

Such thoughts encouraged me to ignore, at least as a worthwhile, temporary exercise, any hunger for immediate, factual proof that Philip Incao was right or wrong in his problematic lecture. Instead, I turned toward the existing literature with a humbler goal in mind: to loosen as best I could those interpretative prejudgements that would blind me to all but the truths surveyed by my own, post-Galenic authorities.

"The heart is not a pump; rather the blood moves the heart!" This seemed to be the rallying cry. It was just bald enough to pique my interest – and to bring me quickly to some curious facts of the literature.

An experiment on toads

The immature, beating heart of the larval orange-speckled toad can be surgically excised, turned 180 degrees around, and reinserted between its artery and vein, yet the direction of blood flow, instead of reversing, remains the same. So at this early stage, before the structures are fully and rigidly developed, the flow shows itself to be primary, determining the heartbeat rather than being determined by it.

The beating heart requires flow

According to one of the classic principles of heart function, known as Starling's Principle, the heart's arterial output is directly modified by (among other factors) the pressure of venous inflow. To this degree, then, the heart is governed by the flow coming to it.

Moreover, without any flow at all, the heart fibrillates – that is, beats chaotically and ineffectually. In order to function normally, the heart must have fluid moved through it. Researchers have made a human heart beat again as late as twenty hours after death by supplying an inflow of blood. (During the interim the heart tissues must be perfused with blood so that they do not die.) Rabbit hearts can similarly resume beating several days after death.

Arteries are not passive

As shown by embryonic development, the heart is a strengthened and elaborated blood vessel. Not surprisingly, then, the blood vessels, which possess their own layer of smooth muscle, share in the rhythmical function of the heart. The pulse felt in the arteries has been found to be an active and resonant one, not simply the passive effect of a pressure pulse issuing from the heart.

Circulation without Heart Movement

In experiments with dogs, the animals were asphyxiated. A half hour after the hearts stopped – when the oxygen content of the arterial blood had fallen to that of

shaped by the set lines [u, v] according to the mapping:

$$\{(x, y) \mid y = f(x)\} \xrightarrow[\substack{\text{inversion}}]{\substack{\text{polar} \\ \text{reciprocal}}} \{[u, v] \mid v = f(u)\} \tag{3.4}$$

Analogously, the polar-reciprocal inversion of a function in counterspace (in line co-ordinates) results in a function in Euclidean space according to the mapping:

$$\{[u, v] \mid v = f(u)\} \xrightarrow[\substack{\text{inversion}}]{\substack{\text{polar} \\ \text{reciprocal}}} \{(x, y) \mid y = f(x)\} \tag{3.5}$$

3.3 Conversion of line co-ordinates in point co-ordinates and vice versa
A function in line co-ordinates may be transformed into a function in point co-ordinates as follows.
Choose two lines (tangents) of the curve $\{[u, v] \mid v = f(u)\}$ close to each other: $[u, f(u)]$ and $[u + \Delta u, f(u + \Delta u)]$. With (2.18) it can be shown that for the point of intersection of these lines (x, y) we have:

$$x = \frac{\dfrac{f(u + \Delta u) - f(u)}{\Delta u}}{f(u) - u \dfrac{f(u + \Delta u) - f(u)}{\Delta u}} \quad \text{and} \quad y = \frac{-1}{f(u) - u \dfrac{f(u + \Delta u) - f(u)}{\Delta u}} \tag{3.6}$$

Now take of this solution the limit $\Delta u \to 0$. Both tangents will approach each other more and more, and the point of intersection approaches to the curve. If one lets the line $[u, f(u)]$ run through the entire curve, the point (x, y) will also run through the curve. Therefore the function in point co-ordinates can be written in the following parameter representation:

$$\left\{ (x, y) \mid x(u) = \frac{f'(u)}{f(u) - u f'(u)} \quad , \quad y(u) = \frac{-1}{f(u) - u f'(u)} \right\} \tag{3.7}$$

Conversion of a function in point co-ordinates to line co-ordinates goes as follows.
Choose two points of the curve $\{(x, y) \mid y = f(x)\}$ close to each other: $\left(x, f(x)\right)$ and $\left(x + \Delta x, f(x + \Delta x)\right)$. With (2.22) it can be shown that for the join [u, v] of these points we have:

$$u = \frac{\dfrac{f(x + \Delta x) - f(x)}{\Delta x}}{f(x) - x \dfrac{f(x + \Delta x) - f(x)}{\Delta x}} \quad \text{and} \quad v = \frac{-1}{f(x) - x \dfrac{f(x + \Delta x) - f(x)}{\Delta x}} \tag{3.8}$$

Now take for this solution the limit $\Delta x \to 0$. Both points will approach each other more and more and the join [u, v] approaches to the tangent of the curve. If one lets the point $(x, f(x))$ run through the entire curve, the line [u, v] will also run through

the curve. Therefore the function in line co-ordinates can be written in the following parameter representation:

$$\left\{ [u,v] \mid u(x) = \frac{f'(x)}{f(x) - xf'(x)} \quad , \quad v(x) = \frac{-1}{f(x) - xf'(x)} \right\} \tag{3.9}$$

Two examples for illustration.

As a first example a curve determined by a function in point co-ordinates:

$$\left\{ (x,y) \mid y = \sqrt{x} \right\} \tag{3.10}$$

From (3.9) we have:

$$u(x) = \frac{\dfrac{1}{2\sqrt{x}}}{\sqrt{x} - \dfrac{x}{2\sqrt{x}}} = \frac{1}{x}, \quad v(x) = \frac{-1}{\sqrt{x} - \dfrac{x}{2\sqrt{x}}} = \frac{-2}{\sqrt{x}} \tag{3.11}$$

Elimination of the parameter x results in the function in line co-ordinates which determines the same curve:

$$\left\{ [u,v] \mid v = -2\sqrt{u} \right\} \tag{3.12}$$

As a second example a curve determined by a function in line co-ordinates:

$$\left\{ [u,v] \mid v = (u-1)^2 \right\} \tag{3.13}$$

From (3.7) we have:

$$x(u) = \frac{2u-2}{(u-1)^2 - (2u^2 - 2u)} = \frac{-2}{1+u}$$

$$\tag{3.14}$$

$$y(u) = \frac{-1}{(u-1)^2 - (2u^2 - 2u)} = \frac{1}{u^2 - 1}$$

Elimination of the parameter u results in the function in point co-ordinates which determines the same curve:

$$\left\{ (x,y) \mid y = \frac{x^2}{4x+4} \right\} \tag{3.15}$$

It must be noted that the examples given are exceptional because in most cases it will not be possible to eliminate the parameter from equations (3.8) and (3.11).

But this acceleration soon plays itself out in the tissues. And if it weren't for the activity of the life field of the sun sphere in our body in the periphery, the blood would pool in the tissues and not be able to overcome gravity and return to the heart.

Shifting the Light

One thing seemed certain: this physician was trying to gaze back across the threshold that Harvey had crossed. But could his strange, archaic-sounding thoughts survive translation into the language of science? How could one go about attempting such a translation? More broadly, is it the unorthodox researcher's main task to hit upon some startling discovery that vindicates his misunderstood views, or rather to help his listeners look upon the existing facts with new eyes?

The historian Herbert Butterfield expressed little doubt about the answer to this last question. Change, he says, was brought about during the scientific revolution "not by new observations or additional evidence in the first instance, but by transpositions that were taking place inside the minds of the scientists themselves." The challenge lay in "the art of handling the same bundle of data as before, but placing them in a new system of relations with one another."

"The supreme paradox of the scientific revolution is the fact that things which we find it easy to instil into boys at school...things which would strike us as the ordinary natural way of looking at the universe...defeated the greatest intellects for centuries, defeated Leonardo da Vinci and at the marginal point even Galileo, when their minds were wrestling...with these very problems. Even the greatest geniuses who broke through the ancient views in some special field of study – Gilbert, Bacon and Harvey, for example – would remain stranded in a species of medievalism when they went outside that chosen field. It required their combined efforts to clear up certain simple things which we should now regard as obvious to any unprejudiced mind, and even easy for a child."[9]

Butterfield notes the dangers in his own field: the student of history, confronted with a huge mass of data, has a kind of magnet in his mind that, unless he is careful, perceives just those elements that fill out the picture he has already formed. Similarly in the sciences: during the centuries immediately preceding Harvey, "though dissection was being practised to an increasing degree it produced little result – men only observed the things which the ancient writer Galen had taught them to look for."

Much of the dissection at the dawn of the age of science was for demonstration before a class, not for experiment and discovery. Its whole purpose was to make visible what Galen had seen:

These poor creatures knew that Galen was a much greater artist in the work of dissection than they would ever be, and they took no end of pride in themselves if the result came out as Galen said it ought to do – a matter not at all easy, especially as Galen used apes, for example, instead of human bodies when he conducted his dissections.

Many a modern student in the dissecting laboratory will recognise the truth of these words! It's a difficult dilemma to escape, since the student who would learn quickly

Incao no longer seemed to inhabit the world of modern science. What was one to make of it? The fullest clue he offered to his overall line of thought was contained in repeated references to "peripheral forces." The blood's primary movement, he declared, arises from the periphery rather than the center. But what this periphery amounted to was hard to grasp. He spoke of "the whole cosmos streaming into us and converging upon our heart, from which it is reflected out again as spiritual light." He pictured how the springtime sun, embracing the redwood tree, raises the sap to a height of two hundred feet – without any benefit of a pump. He quoted Harvey, who likened the blood circulation to the cycling of water in nature:

For the moist earth warmed by the sun develops vapours, the rising vapours again densify to rain and fall downward again, moistening the earth. Here we have the circulation of the sun.

Applying the analogy in a way that Harvey did not, he noted that "in the vast network of capillaries in our bodies, the surface area of the blood is so enormous that it is able to achieve a near weightless state, like mist or fog in the air."

Not waiting for me or anyone else in the audience to brand his thought "medieval," Incao took the label upon himself by adverting to the hoary notion of levity. Referring to the rising and falling of the blood with heat and cold, anger and shame, he related heavy-heartedness to a genuine heaviness of the blood, and light-heartedness to a lightness of the blood. Lightness, he went on, is not merely the absence of gravity; it is an aspect of life. The sap rises in the spring just as the young animal or human being rises up on its legs, opposing gravity and achieving lightness and freedom of movement. But this rising is always accompanied by a falling: the plants fall back to earth in the autumn, we ourselves lie down to sleep at night after rising in the morning, our bodies eventually descend to a stooped and shrunken posture, and we lose the lightness in our steps as we get old and approach death.

So, in the rising and falling of the blood the human being in ancient times felt himself to be participating in the great rhythm of all nature – part of a great cosmic dance between sun and earth. Heaviness was of the earth, and lightness as well as light and life itself, was of the sun....All life does indeed come from the sun, through the dance of the sun's light and lightness with the heaviness of the earth. The Latin word for gravity also means pregnancy. In the womb new life approaches the heaviness of earth. But in order for life to develop, earth's gravity must be offset somewhat by the sun's lightness, by buoyancy, and that's why all animal and human life develops in the fluid-filled womb or the fluid-filled egg.

And what is true of the embryo remains true for the fluid-immersed cells of the body. At the level of the cell, where an almost inconceivably extensive network of capillaries determines the nature of the circulation, the law of the sun, of levity, prevails. Motion arises here as it does in the sap of the redwood, and the movement from thousands of miles of capillaries converges from all sides upon the heart, through the veins, bringing the heart into movement. This primary sun-movement of the blood provokes an answering beat of the heart, which transiently accelerates the blood in its course.

4. Distance in counterspace

By polar-reciprocal inversion w.r.t. an imaginary unit circle of the usual procedure for determining distance, an analogous procedure is generated for determining distance in counterspace. In the next section, the left column shows the usual procedure for determining the distance between two points. The right column shows the polar-reciprocal inverse if this, that is the determination of the distance between two lines in counterspace.

The lines a: $[u_a, v_a]$ and b: $[u_b, v_b]$ in figure 4.2 are the polar-reciprocal inverses of respectively the points A: (x_A, y_A) and B: (x_B, y_B) in figure 4.1.
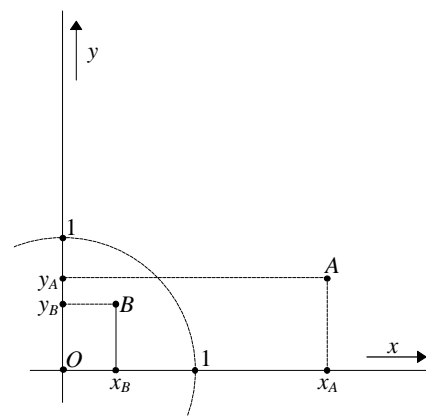
Determining distance in Euclidean space **Determining distance in counterspace**

Figure 4.1

Figure 4.2

The distance between A and B in Euclidean space is defined by:

$$d_{A,B} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \quad (4.1)$$

The distance between a and b in counterspace is defined by:

$$d_{a,b} = \sqrt{(u_a - u_b)^2 + (v_a - v_b)^2} \quad (4.2)$$

In figure 4.1, for instance, the distance between A: (2, 0.7) and B: (0.4, 0.5) is:

$$d_{A,B} = \sqrt{(2-0.4)^2 + (0.7-0.5)^2} = 1.61 .$$

The line a in figure 4.2 is the polar-reciprocal inverse of the point A in figure 4.1 and therefore has line co-ordinates [2, 0.7]. The line b is the polar-reciprocal inverse of the point B and therefore has line co-ordinates [0.4, 0.5]. The distance between a and b equals the distance between A and B: $d_{a,b} = \sqrt{(2-0.4)^2 + (0.7-0.5)^2} = 1.61 .$

5. Surface area in counterspace

By polar-reciprocal inversion w.r.t. an imaginary unit circle of the usual procedure for determining the magnitude of a surface area, an analogous procedure is generated
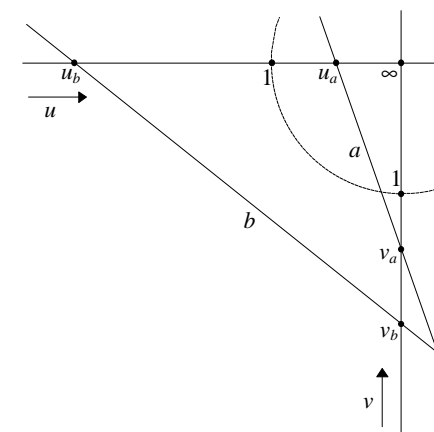
for determining the magnitude of a surface area in counterspace. In the next section, the left column shows the usual procedure for determining the magnitude of a surface area. The right column shows the polar-reciprocal inverse of this.
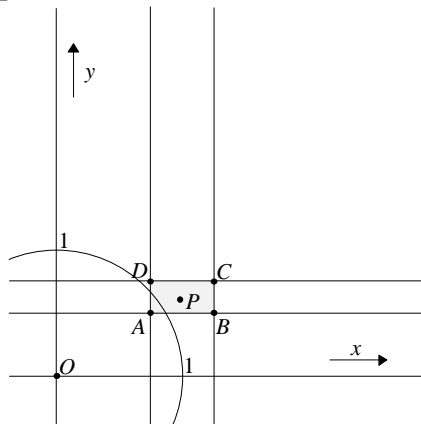
Determining surface area in Euclidean space

Figure 5.1

The surface area to be determined (grey) is fixed by the points *A*, *B*, *C* and *D* and an arbitrary point *P* within this area. It consists of the set of points which are generated by translation of *P* without crossing one of the lines *AB*, *BC*, *CD* and *DA*.

The magnitude of the surface area considered in Euclidean space is determined by the product of the distance between the points *A* and *B* and the distance between the points *B* and *C* in Euclidean space. With use of (4.1):

$$S = (x_B - x_A)(y_C - y_B) \quad \textbf{(5.1)}$$

points *A*, *B* and *C*, we have:

$$u_a = x_A, \quad u_b = x_B, \quad v_b = y_B, \quad v_c = y_C \quad \textbf{(5.3)}$$

Substitution of this result in (5.2) results in (5.1), from which it follows that:

$$S = S^- \quad \textbf{(5.4)}$$

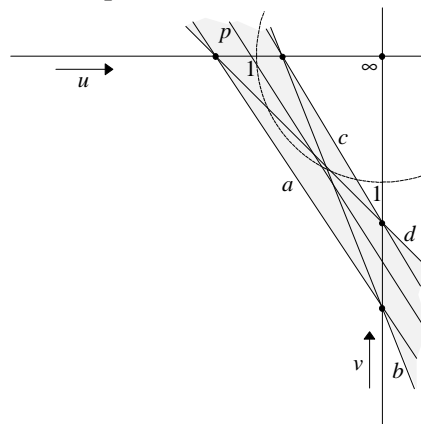**Determining surface area in counterspace**

Figure 5.2

The surface area to be determined (grey) is fixed by the lines *a*, *b*, *c* and *d* and an arbitrary line *p* within this area. It consists of the set of lines which are generated by the rotation of *p* without passing one of the points *ab*, *bc*, *cd* and *da*.
The magnitude of the surface area considered in counterspace is determined by the product of the distance between the lines *a* and *b* and the distance between the lines *b* and *c* in counterspace. With use of (4.2):

$$S^- = (u_b - u_a)(v_c - v_b) \quad \textbf{(5.2)}$$

Because the lines *a*, *b* and *c* are the polar-reciprocal inverses of respectively the

to imagine the circulatory system from a red blood cell's viewpoint. It is a life of ceaseless, bustling movement. Surging into the aorta from the heart, the cell jostles together with uncounted billions of its neighbours, travelling with surprising speed through wide, dark tunnels whose walls suddenly expand and then contract again every second or so. The cell itself vibrates, expanding and contracting two hundred times per minute.

Progressively – never allowed to stop and rest – our red blood cell finds its way into narrower and narrower tunnels, until finally it is constrained by a channel that might be narrower than its own body. Thanks to its donut shape and its inherent powers of mobility, it squeezes and elongates itself to conform to this capillary vessel, pressing on through. The fluid in which it swims is meanwhile passing with remarkable freedom through the walls of the vessel into the surrounding tissues, each cell of which consequently floats nearly weightless in a nutritive sea. At the same time, other fluids pour into the vessel from the outside, accompanying the red blood cell along its way.

Then the channels begin to widen again. Life might seem easier, and yet the cell is growing blue from tiredness. It also has to travel steeply uphill, but somehow it never stops moving. And just when it appears it can go no further, it finds itself in a huge room with doors at the far side. The doors suddenly slam shut, and a moment of rest seems to have arrived at last. But almost immediately the doors open again and something like a great wind, a tremendous tornado, sweeps the cell and its neighbours through the open door and down into the next room, where they are hurled against the round sides of the room. Sliding around these sides, they are then swirled in a great vortex toward the lungs where our adventuresome cell finally "gets some fresh air" and feels full of life again.

Periphery and Center

Till now, Incao – who is an M.D. and general practitioner – had followed conventional scientific understandings, even if his point was that our imaginations, ever willing to rest in death-like immobility upon structures and mechanisms, usually belie those understandings.

But then his argument turned downright medieval. "We shouldn't ask why the blood moves," he said. The question betrays a false picture, a picture of death, not life. The blood moves because it is inherently alive, and it is the property of all life to move. Blood that needs a heart to pump it is dead blood." He then offered us a series of statements, almost in verse form:
** Spirit precedes matter.
** Life precedes death.
** The whole precedes the part.
** Movement precedes stillness.
And if we really believe this, he said – if we are capable of seeing it – then we will realise that
** The heart is not a pump; rather the blood moves the heart!

So, again, we observe a poet seeking the living heart. But now, thanks to the attempted re-engineering of Harvey's "piece of machinery," the search is more urgent and the motive more impelling than ever before, even if the goal remains discouragingly obscure. Can we discover a unity between the heart of the poet and the heart of science, or must we forever accept a heart broken between two incommensurable worlds?

It is not only the poet who sends us upon our search. Today we are urged from many sides to rise above mechanism by making our thinking holistic, organic, or even spiritual. Yet, for all the value in some of these efforts, one wonders whether the trillion-dollar juggernaut of institutional science has been deflected more than a degree or two by the new movements – many of which, it must be admitted, sorely lack the rigor, efficacy, and objectivity of genuine science.

If there is one place above all others where we might demand a clear demonstration of explanatory power by the alternative approaches, it is in our understanding of the human heart. But this power must be real and down to earth; it must, as Hamsun told the frantic grandmother, be a matter of life and death.

The Movement of the Blood

I sat in the small lecture hall with my eyes closed, trying to fulfil the speaker's charge. "Visualise the human circulatory system in its fullest reality," he had instructed us. Those several years ago I did not even remember the nature of the major divisions of the heart, let alone the primary pathways leading to and from it. But I knew enough to summon a vague, anatomy-book image of the central heart, with red arteries emerging and branching out to all parts of the body, and blue veins returning to the heart. I also knew that capillary networks joined the arteries and veins. That was about as far as I got with my imagining; I didn't even have the presence of mind to start the heart beating.

My view of the circulatory system, it turned out, was not atypical, at least not so far as the members of this particular audience were concerned. The lecturer, Philip Incao, seemed unsurprised. We had, he pointed out, visualised the relevant structures and mechanisms quite well. But how much of the truth of the circulatory system is discernible from these static entities? He answered by offering us another sort of picture.[8]

The human organism, Incao told us, begins life as streaming protoplasm. In the young embryo, blood cells form and move first; then some of those cells lengthen and shape themselves into vessels; and finally, from a swollen portion of blood vessel, the heart is formed. Structures take shape only as a kind of condensation or filling in of the forms first established by streaming movement (rather, I thought, as if the energies of an invisible dance, over time, were to precipitate the perfect material dancer for that dance – a dancer who was the very embodiment of the dance). He cited the German poet, Novalis, to the effect that the human body is a formed stream.

The body solidifies and hardens with time – until rigidification finally produces death. But so long as it lives, movement and fluidity remain primary. Incao asked us

Thus, the magnitude of a surface area equals that of the polar-reciprocal inverse of this area in counterspace. This result is in agreement with the fact that by polar-reciprocal inversion, each point within an area is transformed one-to-one into a line.

For instance the determination of the magnitude of the surface area in counterspace of the quadrangle between the lines a: [0.75, 0.5], b: [1.25, 0.5], c: [1.25, 0.75] and d: [0.75, 0.75] (see figure 5.2). Substitution in (5.2) gives:
$$S^- = (1.25 - 0.75)(0.75 - 0.5) = 0.125 .$$

6. Differentiating in counterspace

By polar-reciprocal inversion w.r.t. an imaginary unit circle of the procedure for determining the derivative of a function in Euclidean space, an analogous procedure is generated for determining the derivative of a function in counterspace. In figure 6.1 both procedures are shown.
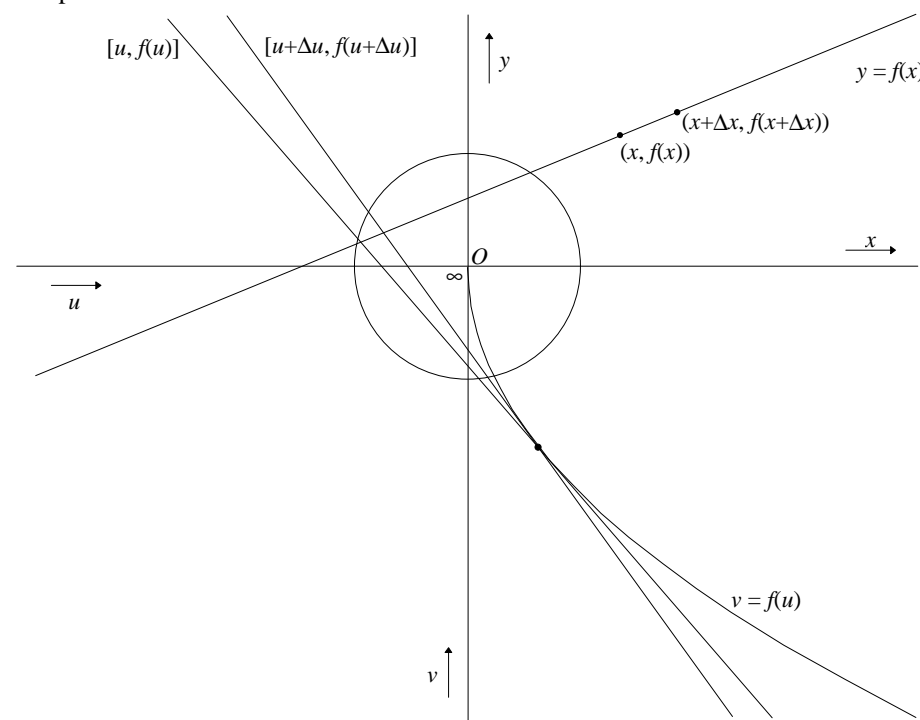

Figure 6.1

In the first quadrant of figure 6.1, a function is shown in point co-ordinates: $y = f(x)$. The curve in the third quadrant is the polar-reciprocal inverse of this function, that is the same function but this time in line co-ordinates: $v = f(u)$. Compare with figure 3.2; in figure 6.1 the tangents of the inverse function are left out for clearness.

In the next section, the left column shows the usual procedure for differentiating a function in Euclidean space. The right column shows the polar-reciprocal inverse of this, that is the procedure differentiating a function in counterspace.

Differentiation in Euclidean space
Given a curve in point co-ordinates in Euclidean space:

$$\{(x, y) \mid y = f(x)\}$$

Choose two points on this curve, close to each other: $(x, f(x))$ and $(x+\Delta x, f(x+\Delta x))$ The derivative of the function is defined as:

$$\frac{df(x)}{dx} \equiv \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad (6.1)$$

From (3.9) it follows that for the tangent $[u, v]$ in the point $(x, f(x))$ of the curve:

$$\frac{u}{v} = -f'(x) \quad (6.3)$$

The derivative adds to every point (x, y) of Euclidean space a tangent $[u, v]$. The co-ordinates u and v result from substitution of (6.3) in (2.15):

$$u = \frac{f'(x)}{y - xf'(x)}$$
$$\quad (6.5)$$
$$v = \frac{-1}{y - xf'(x)}$$

As it were, Euclidean space is 'shaped' by the lines of counterspace. This is the 'direction field'. An infinite number of functions in point co-ordinates 'fit' in this direction field. (This is expressed by the integration constant when integrating $f'(x)$).

Differentiation in counterspace
Given a curve in line co-ordinates in counterspace:

$$\{[u, v] \mid v = f(u)\}$$

Choose two lines (tangents) of this curve, close to each other: $[u, f(u)]$ and $[u+\Delta u, f(u+\Delta u)]$. The derivative of the function is defined as:

$$\frac{df(u)}{du} \equiv \lim_{\Delta u \to 0} \frac{f(u + \Delta u) - f(u)}{\Delta u} \quad (6.2)$$

From (3.7) it follows that for the point of contact (x, y) of the line $[u, f(u)]$ to the curve:

$$\frac{x}{y} = -f'(u) \quad (6.4)$$

The derivative adds to every line $[u, v]$ of counterspace a point of contact (x, y). The co-ordinates x and y result from substitution of (6.4) in (2.15):

$$x = \frac{f'(u)}{v - uf'(u)}$$
$$\quad (6.6)$$
$$y = \frac{-1}{v - uf'(u)}$$

As it were, counterspace is 'shaped' by the points of Euclidean space. This is the 'direction field'. An infinite number of functions in line co-ordinates 'fit' in this direction field. (This is expressed by the integration constant when integrating $f'(u)$).

take the models, whether geometrical or mechanical, literally....It was soon to be stamped indelibly on men's imaginations by the circumstance of their being ever more and more surrounded by actual artificial machinery on earth."

William Harvey stood in the midst of this transition. It was in 1633, seven years after the publication of The Motion of the Heart, that Galileo kneeled in Rome to recant his Copernican views. We can begin to suspect, then, that Harvey's achievement, too, may have had less to do with the demonstration of new facts – although it certainly had to do with that – than it did with an assertion about what sort of fact could throw a penetrating and true light upon the human being. And his contention was that mechanical facts could do so.

So far as we know, Harvey was the first person to conceive the heart in such a fashion. Apparently, one simply could not imagine a mechanical heart during earlier centuries. In our time, by contrast, it is nearly impossible to find a discussion of the heart, whether popular or scientific, that does not mention "the body's marvellous pump." Yet even now it is nearly impossible for us to sustain the mechanical habit of thought when we face our own bodies. The heart's "skipped beat" presents itself to immediate experience more as a tiny seizure of unease than as an equipment glitch. And nothing rouses in us a more primal fear than a sudden bout of arrhythmia or tachycardia (wildly accelerated heart).

Following his own, late-night resort to an emergency cardiologist, the poet Charles Siebert noted the consensus among physicians that "the best way to cure this fear for one's heart is to instruct patients to look at the heart mechanically, technically – to, in a sense, dehumanise it. If the heart's a pump, there is nothing to worry about. We make those. They're simple, long-lasting, and easily repaired – even replaced, at least in part – when broken."

Siebert was moved by his own crisis to investigate the badly failed experimental implants of the plastic and titanium Jarvik-7 heart back in the 1980s. Quite apart from the systematic breakdown of other organs following the implants ("it's as though the organs asked for more from the heart than an efficient, robotic output, asked for some subtlety and variety of pulse, a virtuosity the Jarvik-7 was simply not capable of"), Siebert asked himself about the peculiar psychological symptoms of these patients:

How it actually felt to be alive with a monorhythmic piece of plastic clicking where once the mime – if not the author – of our emotions dwelled is a secret the recipients have taken with them. But I've always wondered if this depression, this loss of aspects of their personalities, had anything to do with a proportionate loss of aspect in their new heart's response to the varying emotional stimuli around them. Could [Barney] Clark's or [William] Schroeder's depression have stemmed not only from physical suffering but also from the severance of that natural conversation between the heart and the brain – a disparity they recognised, say, between the way their natural hearts may have once reacted when they saw their wives walk into a room and the way their new ones couldn't in that same instance?[7]

neither the only, nor probably the first astronomer to think of it. Copernicus himself knew this. Secondly it is generally believed that the Church tried to keep the discovery dark. Actually Copernicus did not himself want to publish his De Revolutionibus Orbium, and was only eventually prevailed on to do so by the importunity of two eminent Churchmen.

Barfield explains further that the decisive turning point in the history of astronomy occurred, not because of the unoriginal Copernican hypothesis, nor because many people came to accept the hypothesis as a reasonable one, but rather because investigators like Kepler and Galileo "began to affirm that the heliocentric hypothesis not only saved the appearances, but was physically true." In other words, "it was not simply a new theory of the nature of the celestial movements that was feared, but a new theory of the nature of theory; namely, that, if a hypothesis saves all the appearances, it is identical with truth."

Deriving geometrical hypotheses that "saved the appearances" of the celestial motions was a respectable pursuit of long standing – the Ptolemaic system itself was taught in the same way, not as the underlying truth of the matter. "It worked," as we might say today with full scientific respectability. The Church was quite willing to accord similar status to the Copernican hypothesis – and even to say, "it works better."

All this is hard for us to stomach.

When the ordinary man hears that the Church told Galileo that he might teach Copernicanism as a hypothesis which saved all the celestial phenomena satisfactorily, but "not as being the truth," he laughs.[6]

But laughter, like Guyenot's charge of stupidity, is risky. What are we missing?

Interestingly, contemporary physics presents us with troubling questions about the status of our own hypothetical models. The orthodox view today – despite our inability to live up to it – is precisely that the models must not be mistaken for reality. Our light behaves as if it were a wave and as if it were a particle, but it is neither – certainly not in any literal, picturable sense. This has been a painful pill for us to swallow – and it is not clear that we have yet succeeded in the swallowing. Apparently the movement in the opposite direction – to accept a mechanical, explanatory model as equivalent to reality – was just as painful for Galileo's opponents.

Our own science, then, is strongly encouraging us to look back across the threshold of the scientific revolution for possible relief of our perplexities. Did those who resisted the arguments of Galileo – and those who stumbled at the extraordinary idea that the heart is really a machine – instinctively sense the difficulties that have come to haunt the most advanced researchers today?

The Human Machine

Years before Galileo, as Barfield reminds us, the Arabs, following the Ptolemaic hypothesis, had constructed mechanical models of the solar system for purposes of calculation. The modern understanding of the world was born "when men began to

From equations (6.1) and (6.2) it follows that differentiation in counterspace obeys the same rules and has the same standard derivatives as differentiation in Euclidean space. For example the curve determined by the function in point co-ordinates: $\left\{ (x, f(x)) \mid f(x) = \sqrt{x} \right\}$. The derivative of this function is: $f'(x) = \dfrac{1}{2\sqrt{x}}$.

The *same* curve in counterspace (that is, expressed in line co-ordinates with help of (2.9)) is: $\left\{ [u, g(u)] \mid g(u) = -2\sqrt{u} \right\}$. The derivative of this function is: $g'(u) = -\dfrac{1}{\sqrt{u}}$.

Figure 6.2 shows the function f (or in line co-ordinates: g) with the accompanying direction field that is determined by $f'(x)$ (or in line co-ordinates: $g'(x)$).
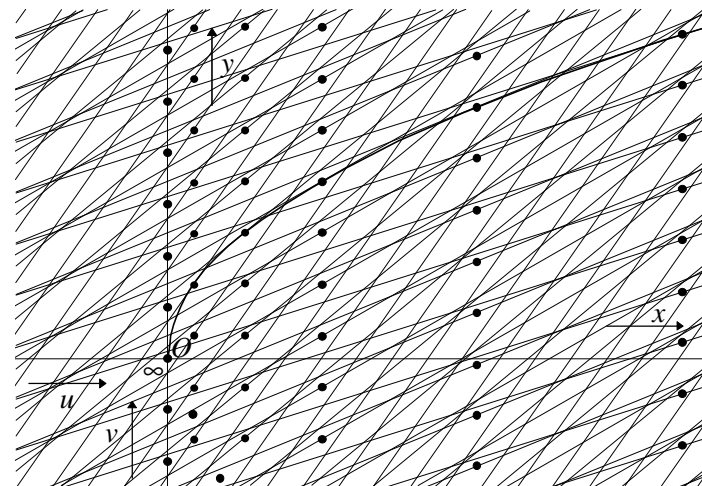
Figure 6.2

Through every point of contact in the direction field goes exactly one tangent. From (6.5) it follows that for the tangent $[u, v]$ in the point (x, y) of the direction field we have: $u = \dfrac{1}{2y\sqrt{x} - x}, \quad v = \dfrac{-2}{2y - \sqrt{x}}$. For example: through the point $(1, 1)$ goes the line $[1, -2]$.

7. Integrating in counterspace

Figure 7.1 shows how the procedure of integrating the function $y(x)$ can be polar-reciprocally inverted to obtain an analogous procedure for integrating a function in counterspace.

Figure 7.1

By polar-reciprocal inversion w.r.t. an imaginary unit circle of the function $y = f(x)$ the points of the surface below $f(x)$ transform in lines that envelope the inverse function $v = f(u)$. In this way in figure 7.1 the marked 22 points below $y(x)$ in the area indicated in grey bet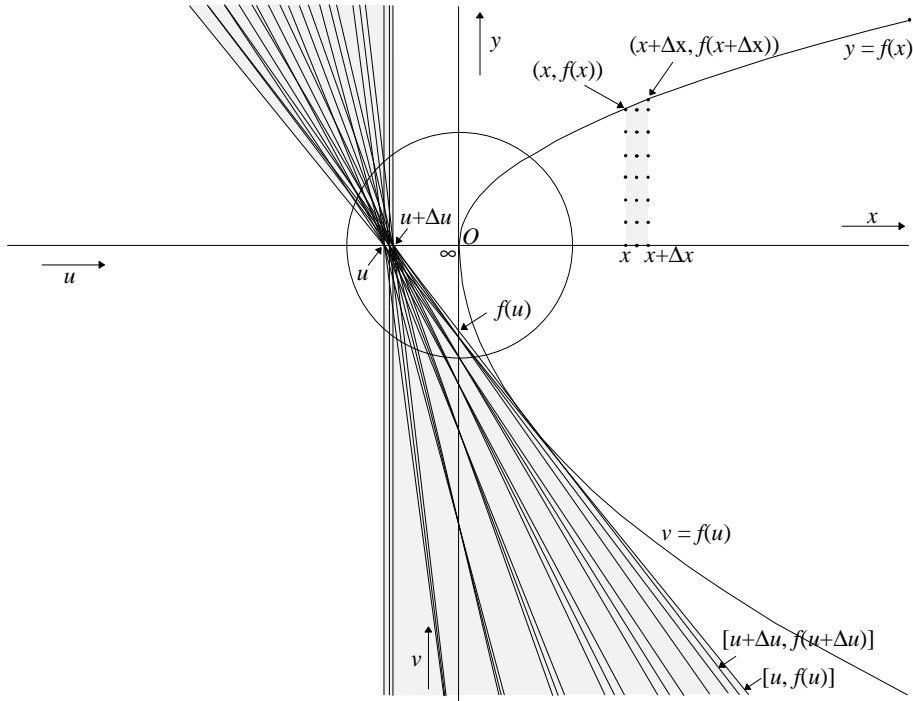ween x and $x+\Delta x$, transform in 22 lines in the area between u and $u+\Delta u$, also indicated in grey. The area last mentioned touches the inverse function, analogously to how the area first mentioned touches the original function.

Further, a careful observation shows that the border of the grey area against the inverse function is not a straight line but bends somewhat around it. Therefore this grey area is not an exact quadrangle. This is the result of the fact that the grey area below the original function is not an exact quadrilateral.

For the magnitude of the grey area between x and $x+\Delta x$ we have:

$$\Delta S \approx f(x)\Delta x \qquad (7.1)$$

For the magnitude of the grey counterspatial area between u and $u+\Delta u$ we have according to (5.2):

$$\Delta S^- \approx (u + \Delta u - u)(f(u) - 0) = f(u)\Delta u \qquad (7.2)$$

The magnitude of the area below the function $y(x)$ between x_1 and x_2 is:

Perhaps, inclined to accuse van den Berg of playing word games, you are adamant: "Regardless of whether physicians had previously thought of the heartbeat as the sound of a pump, what they heard was nevertheless a pumping sound." But that is to miss the point. Parisano could have no hope of winning his debate on the strength of a transparent word game. He did not say, absurdly, "No one ever thought to liken the heart's sound to a mechanical beating before, so the heart doesn't make a beating sound." Rather, he said, "the heart obviously does not beat; it does not in fact make any sound that could be likened to a mechanical beat" – and he spoke in the manner of those who have incontrovertible fact on their side.

So here, precisely at the threshold, we are met with a nearly incomprehensible datum: a physician and anatomist, doubtless eager to find the most effective means to discredit Harvey before the general public and the medical profession, appeals confidently to an empirical fact, available to everyone, that refutes Harvey's claims.

Perhaps, at least, we can now understand Harvey's delayed publication a little better. Every genuine revolution in understanding is at first foolish and obscure – precisely because it is a revolution in understanding, and therefore is not readily accessible to previous modes of thought and experience.

Putting it a little differently: what we are capable of perceiving is determined by what we are capable of conceiving, and the great discoverer – by thinking ahead of his contemporaries – perceives what remains inaccessible to them.

But surely Parisano heard something, for he possessed ears. Why does his hearing escape our ears? Are we as benighted as Parisano was when it comes to rising above the provincialism of our own age? He could not hear the heart's beating amid all its other voices; we, on our part, can hear nothing but the beating. Our backward gaze is as uncomprehending as Parisano's forward gaze – this despite the fact that we might expect our own past to be more reliably accessible to us than the future was to him.

So perhaps we should admit: No, we can't understand Harvey's achievement, since his achievement consisted in overcoming the force of contemporary objection, and that objection seems laughably empty to us.

What Is Scientific Truth?

Harvey's public lecture of 1616 occurred only seven years after Galileo first turned his newly constructed telescope upon the heavens, where he observed the moon's pockmarked surface, the phases of Venus, the moons of Jupiter, and the sun's darkened blemishes. As Romanyshyn notes, these observations would contribute heavily to the immediate, psychological reality of the still much-disputed Copernican heliocentric outlook.

This ferment and discovery, repeated in so many fields of inquiry, was characteristic of the scientific revolution – which is the broadest name for the threshold we have been considering. Of the central element in the revolution, Barfield writes:

The popular view is, that Copernicus "discovered" that the earth moves round the sun. Actually the hypothesis that the earth revolves round the sun is at least as old as the third century B.C., when it was advanced by Aristarchus of Samos, and he was

employ as diverting interludes before they get back to the clear-headed, hard-nosed business at hand. We must, after all, be honest: the juxtaposition was hardly an equal one. Harvey fathered medical science; Hamsun was a poet, and even if he really had a clear idea about what he was doing with the stricken boy, he didn't let on. What is to be gotten from such a lopsided comparison?

So let us by all means stick with William Harvey. But if we are to understand his achievement, we must first look across the threshold that he looked across – except in our case the only way to look across it is in the opposite direction and from very far away. Harvey's boldness is our conventional wisdom. That is the difficulty. Hamsun's memory – of unknown provenance and content – may be impotent to point the way, but we must find some means to gaze back into the world from which Harvey emerged, bringing it alive. Otherwise, we can only applaud Harvey for saying the obvious – hardly a matter of historic note.

One way to become aware of the ground covered in a historic step forward is to look back and see, if we can, how the step appeared to those who refused it.

Harvey drew the attention of his readers to the heart's beating. "With each motion of the heart, when there is a delivery of a quantity of blood, a pulse can be heard within the chest." The Dutch historical psychologist, J. H. van den Berg, drawing on a history of the sounding heart[2] as well as his own extensive researches, tells us that "Harvey's view met with no approval. No one in the field of medicine from the time of Hippocrates to William Harvey ever heard the heart beat, and no colleague of Harvey showed any inclination to follow him in this unexpected and strange point." Van den Berg goes on to cite a pamphlet against Harvey by Emilio Parisano, a physician and anatomist in Venice, published in 1635:

Nobody in Venice ever heard the heart beat, and nobody will be able to know what Harvey means until the time comes when he is inclined, which nobody expects, to lend us his ears.[3]

The French historian, E. Guyenot, noting this objection, declared that evidently "human stupidity knows no bounds." [4] Nearly everyone today would agree. But such dismissal is always dangerous; it easily conceals a refusal to stare into the strangeness lying across a historical threshold. Such, in fact, is the case here, for "there can be no doubt that not one physician before Harvey mentioned the sound of the heartbeat." Van den Berg is dead serious. No physician before Harvey mentioned the heart's beating. And yet, the heart's audible sound is discussed freely by various authors of religious and artistic literature. How, van den Berg asks, can this puzzle be understood?

The answer is that nobody before Harvey saw the heart as a pump, with the consequence that nobody, particularly no physician, was able to hear the heart beat as a pump. But this is exactly what William Harvey did. He saw and he heard the heart as a pump. The religious and artistic writers discussed the heart in a different, non-medical context...and that context allowed them to hear the heart: to hear the whispering, wailing, loving, longing tale of that center of the human body.[5]

$$S = \sum_{x_1}^{x_2} \lim_{\Delta X \to 0} f(x)\Delta x = \int_{x_1}^{x_2} f(x)dx \tag{7.3}$$

Analogous, we have for the magnitude of the counterspatial area that envelopes the inverse function $f(u)$ between u_1 and u_2:

$$S^- = \sum_{u_1}^{u_2} \lim_{\Delta u \to 0} f(u)\Delta u = \int_{u_1}^{u_2} f(u)du \tag{7.4}$$

For example the curve determined by the function in point co-ordinates:

$$\left\{ (x, f(x)) \mid f(x) = 2\sqrt{x} \right\} \tag{7.5}$$

The integral of this function is:

$$S = \int_{x_1}^{x_2} 2\sqrt{x}\,dx = \tfrac{1}{3}(x_2^{3/2} - x_1^{3/2}) \tag{7.6}$$

The *same* curve in counterspace (i.e. expressed in line co-ordinates) with use of (3.9):

$$\left\{ [u, g(u)] \mid g(u) = -\sqrt{u} \right\} \tag{7.7}$$

The integral of this function is:

$$S^- = \int_{u_1}^{u_2} -\sqrt{u}\,du = \tfrac{2}{3}(u_1^{3/2} - u_2^{3/2}) \tag{7.8}$$

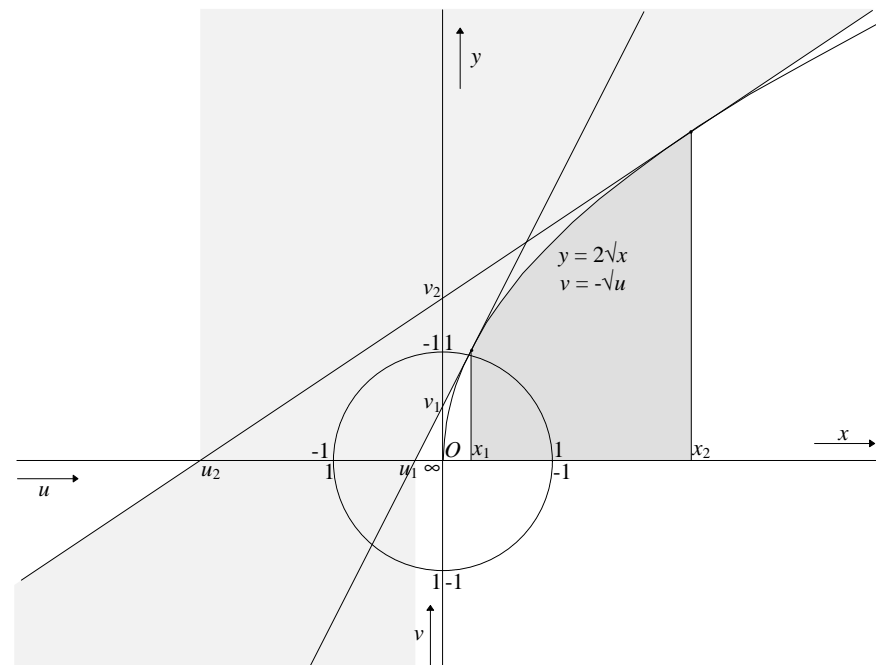Figure 7.2 shows the integrals according to respectively (7.6) (dark grey) and (7.8) (light grey).

Figure 7.2

According to (3.9) we have for the line co-ordinates of the tangents in $\left(x_1, 2\sqrt{x_1}\right)$ and $\left(x_2, 2\sqrt{x_2}\right)$ respectively:

$$u_1 = \frac{1}{x_1}, \quad v_1 = -\frac{1}{\sqrt{x_1}}, \quad u_2 = \frac{1}{x_2}, \quad v_2 = -\frac{1}{\sqrt{x_2}} \tag{7.9}$$

Substitution of (7.9) in (7.8) results in an expression for the counterspatial area below the function, expressed in the co-ordinates x_1 and x_2:

$$S^- = \tfrac{2}{3}\left(\frac{1}{x_1^{3/2}} - \frac{1}{x_2^{3/2}}\right) \tag{7.10}$$

A general equation for the counterspatial area of a function, expressed in Euclidean co-ordinates, can be derived as follows.

Given the function $y = f(x)$. From (3.9) we have for the function $v = g(u)$ that describes the same curve in line co-ordinates:

$$u = \frac{f'(x)}{f(x) - xf'(x)}, \quad g(u) = \frac{-1}{f(x) - xf'(x)} \tag{7.11}$$

From the first part of (7.11) it follows:

$$du = \frac{f(x)f''(x)}{\left(f(x) - xf'(x)\right)^2}\,dx \tag{7.12}$$

Substitution of the second part of (7.11) and (7.12) in (7.4) gives:

$$S^- = \int_{x_1}^{x_2} \frac{-f(x)f''(x)}{\left(f(x) - xf'(x)\right)^3}\,dx \tag{7.13}$$

From this equation it follows that the counterspatial area of constant or linear functions (in point co-ordinates) equals zero. For then the second derivative equals zero.

8. Conclusion

In this paper I have taken the following route to arrive at some concepts of differentiation and integration in counterspace.

1. Euclidean space was recognised as a space wherein the point is the basic element. In Euclidean space lines and curves are described as sets of points. Functions in Euclidean space are expressed in point co-ordinates: (x, y). Contrary to this we have counterspace as a space with the line as basic element. In counterspace curves are described as sets of lines; also a point is interpreted as a set of lines. Functions in counterspace are expressed in line co-ordinates: $[u, v]$.

So the discovery of the circling of the blood, while pointing to the future, also bore testimony to the past. This, of course, is the way with every discovery. Harvey, like all men to one degree or another, lived between two eras. We can assess no human work except as a work of transition.

But we must also attend to the other, practical side of his undertakings. Harvey dissected animals – all sorts of animals: dogs, cats, pigs, serpents, frogs, fish, crabs. And human cadavers. Looking for the secret of the living heart, he was driven by a seemingly inescapable logic toward the dead, dismantled heart. He was discouraged by the dizzying unclarity of the heart's motion during the uncertain interval between incision and death – a motion "which in many animals is accomplished in the twinkling of an eye, coming and going like a flash of lightning." He confessed that "I was almost tempted to think, with Fracastorius, that the motion of the heart was only to be comprehended by God."

But he persevered, "having frequent recourse to vivisections," and paying particular attention to those moments "when the heart begins to flag, to move more slowly, and, as it were, to die." It was only through these repeated attempts to catch the fleeting secret at the moment of its passing – situating himself between life and death – that he finally grasped the essential principles of the circulating machine.

Harvey lived in the passage from one era to another, and the very heart of that transition, displayed in all its earthy, putrefying gore on the tables of his study, was found in the passage from life to death. Living between two eras, might not Harvey have sensed the world-shattering significance of both his methods and his discoveries? And, in this, wasn't there cause for reticence?

When Did the Heart Begin to Beat?

Where William Harvey, glimpsing the age to come, sought the throbbing secret of the living heart by stilling and dissecting it, Knut Hamsun, reminded of an ancient memory, strove to bring a paralysed heart back into life-sustaining motion. Harvey's vision focused incisively upon the heart organ itself, as the central motor of the body. Hamsun's ministrations addressed the heart indirectly, via the blood in the farthest extremity. Harvey discerned a pumping machine, the efficient kick-starting of which would eventually be performed in the most direct manner possible, by compressing the chest or administering a violent shock. Hamsun brought the entire body – and the heart with it – into something like the flowing, rhythmical motion of the heavens.

Both men may have shown a certain reticence at the threshold of mystery, but if they did so, it appears they were straining in opposite directions – Harvey toward the clear, bright, objective light of the laboratory, and Hamsun toward...well, who can say exactly, for if there is a light of the past – if there was reason in the ancient doctrines of vital spirits and mixed blood and the transformation of substance – it is not a reason we can easily appreciate today.

And there, apparently, the tale ends. We are left with a provocative juxtaposition, a properly muted hint of mystery, and a poetic gloss that one occasionally hopes may substitute for profundity – just the formula that many popular books on science

The reader today may be justified in the perplexed feeling that, whatever the truth of the matter, it is now out of reach.

Hearts Not Revived

Two and a half centuries earlier we find another gesture of restraint and reticence toward the beating heart.

In 1628 William Harvey inaugurated the modern science of medicine by publishing his impressive treatise, An Anatomical Disquisition on the Motion of the Heart and Blood in Animals. In this work he traced the blood's heart-impelled circulation. While he could not yet know of the capillaries, he nevertheless demonstrated through painstaking experiment and lucid argument that the same blood travelling out from the heart eventually finds its way back along a closed set of pathways. Among his many achievements, he was the first to recognise that the blood does not flow both ways through its vessels.

The engine of this movement, Harvey wrote, was the heart, "a piece of machinery in which, though one wheel gives motion to another, yet all the wheels seem to move simultaneously." Previously the heart had been charged with heating the blood, transforming its substances, helping to refresh the vital spirits – but not driving the blood throughout the body. With Harvey's revolutionary publication, the study of the heart as a pump was launched, leading finally to the miracle of open-heart surgery assisted by sleek pumping devices that exceed, along certain dimensions of mechanical perfection, anything Harvey could have imagined.

But I spoke of reticence. Harvey had already broached his new understanding of the mechanically pulsating heart twelve years earlier in a lecture. Yet throughout the intervening period he avoided publishing his views to the world at large. Puzzled by this, Robert Romanyshyn tries to imagine the extended interlude:

In these twelve years he will converse with the king [to whom his book would later be dedicated] and become his friend, and he will walk the evening streets of London between his patients' houses and his home, knowing what he has seen and marshalling more evidence on behalf of his daring vision. But in this same period of time he will not raise his voice above a cautious whisper to proclaim his views. Only a select circle of colleagues will hear of his discovery. For twelve years he will remain tentative and perhaps even diffident in the elaboration of his conviction.[1]

Eventually Harvey's colleagues prevailed upon him to publish what he knew.

At a time when politicking for Nobel Prizes achieves distasteful proportions, we may have a hard time understanding this hesitation in a working scientist who had successfully heaved aside some fifteen hundred years of learned inquiry going back to Galen and Aristotle. Nor can we know the causes of his hesitation with any certainty.

But we can at least surmise. The linguistic historian, Owen Barfield, points out that Harvey, sufficiently rooted in the past to assume the existence of vital spirits, consciously based his discovery of the blood's circulation upon two Aristotelian and medieval doctrines: the intimate, interpenetrating relation of the human microcosm to the celestial macrocosm (with the heart ruling the microcosm rather as the sun rules the macrocosm); and, secondly, the perfection of circular movement.

2. Euclidean (point) co-ordinates and counterspatial (line) co-ordinates can be presented in one orthogonal co-ordinate system. Herein x-axis and y-axis have the usual meaning of the Euclidean co-ordinates. The u-axis and the v-axis lie on the x-axis and the y-axis respectively, but have a different measure. We have: $u = -\dfrac{1}{x}$ and $v = -\dfrac{1}{y}$. A line with line co-ordinates $[u_a, v_a]$ intersects the u-axis in u_a and the v-axis in v_a.

3. By polar-reciprocal inversion w.r.t. an imaginary unit circle, Euclidean space is transformed into counterspace. With this, every point with point co-ordinates (p, q) transforms in a line with the same line co-ordinates $[p, q]$. Every curve in point co-ordinates is transformed into an (other) curve in counterspace in line co-ordinates.

4. One and the same curve can be described as a function in Euclidean space in point co-ordinates and as a function in counterspace in line co-ordinates. It has been derived how both representations of the same curve can be converted in each other.

5. By polar-reciprocal inversion w.r.t. an imaginary unit circle of the usual procedure for the determination of distance, an analogous procedure has been derived for the determination of distance in counterspace.

6. With use of the procedure for determination of distance in counterspace, a procedure was derived for the determination of the magnitude of a surface area in counterspace.

7. By polar-reciprocal inversion of the usual procedure for differentiation, an analogous procedure has been derived for differentiation in counterspace. It appears that this amounts to differentiating a function in line co-ordinates. Differentiation of a function in line co-ordinates as well as differentiation of a function in point co-ordinates results in a 'direction field'. This direction field expresses the fact that the derivative of a function attributes to every point of Euclidean space one line of counterspace and vice versa. The derivative of the function determines a relation between Euclidean space and counterspace that, as it were, shapes space.

8. Also by polar-reciprocal inversion of the usual procedure, a procedure has been derived for integration in counterspace. It appears that this amounts to integration of a function in line co-ordinates; by this the area that envelopes the function is determined. It appears that the counterspatial surface of a function amongst other things depends on the curvature of this function. In case of a zero curvature, the magnitude of the counterspatial surface equals zero.

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Between Discordant Eras

Stephen L. Talbott

During the 1880s the young Norwegian poet, Knut Hamsun – who would several decades later receive the Nobel prize for literature – put in time as a transient labourer on the American prairie. In Vagabond, his brief memoir of the period, Hamsun related the story of an accident that befell a small boy named Edwin. While handling some lumber, "Farmer Rodgers" – Edwin's grandfather – lost control of a plank, which struck the boy just above the eye. Edwin fell down "and lay there as if dead."

Hamsun, out on the prairie ploughing, was urgently summoned. He quickly loosed the mules from the plough and ran to the homestead.

"When I came to the farm, both grandparents were in full despair, and there was no end of their wailing. Mrs. Rodgers rolled the child on the floor this way and that, but could not bring him back to life. An ancient memory from youth came to my aid, and suddenly I had no doubt what was to be done. "Take his jacket off," I said. I had put my razor on my bed under the pillow, and now I quickly retrieved it. When I returned, I ripped off Edwin's shirtsleeve and began to cut into a vein of his arm.

The woman gave a shriek and threw herself upon me like one possessed; Alice [the boy's aunt], too, could not be stopped, and said I wanted to kill the child. I shoved her aside with my foot. "It is a matter of life or death; I want to save the child." The old Rodgers recollected himself at these strong words and helped to hold the arm. He only asked, "Can it be good to open the vein?"

When I cut a little deeper, blood came, at first only as a tiny leakage, and then in a fine line. I opened the shirt and listened to Edwin's breast. The heart was silent. Then I seized him by the legs and whirled him, head downward, this way and that. The blood now came in a stream. Then I laid the child down again and listened – the heart beat very softly. That was the most wonderful operation I could have wished for. We all stood there and regarded the child. The small finger on his one hand moved a little. "He just moved his finger," said Mr. Rodgers, half choking for joy. "He moved his finger," the old grandmother said too, and she went sobbing out of the room. Soon the child opened a pair of confused eyes and closed them again. "He opened his eyes!" said Mr. Rodgers. "He's alive.""

Despite serious injury, Edwin survived.

It is a curious story. Where you and I might have shouted, "Pound on his chest!" Hamsun, driven by an ancient memory, bled the boy and whirled him around. He sought first, it appears, to get the blood moving in the hope that this would bring the forgetful heart back into dutiful motion.

But with a poet's restraint and delicacy, Hamsun refuses to speak directly of his inspiration. Having received his memory with faith, and having staked a young boy's life upon the memory's truth, he was content to leave the episode behind a translucent veil.

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Progress towards complementarity in genetics

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Summary
The appearance of adaptive mutations in bacteria raises basic questions about the genetic theory of spontaneous mutation and hence the concept of the generation of biological variation. Adaptive mutations were observed in bacteria exposed to selective conditions during the stationery phase of growth in the absence of DNA replication. Both anabolic and catabolic traits were affected. None of the classical explanations, which depend on errors and irregularities during the replication process, is able to account for these mutations. Various observations suggest new mechanisms for the generation of genetic variation. The theory of adaptive mutations paves the way for the introduction of complementarity in modern genetics.

Theories of adaptive mutations elaborated before the era of molecular genetics argue strongly for holistic approaches to life and heredity. They make a revision of the current concepts of reductionist biology necessary. A synthesis is presented that considers the function of spontaneous as well as adaptive mutations in the development and evolution of organisms. Both forms of mutations reflect the fundamental quality inherent among all living beings; i.e. self-relation and world-relation.

Introduction
According to modern theories of heredity and evolution the tremendous variation amongst living organisms comes about in two ways, namely through spontaneous mutation and through chance hybridisation during sexual reproduction. An overwhelming number of publications provides evidence for chance variation. Because of this chance variation and DNA molecular replication (doubling) processes, which produce changes in the genetic make-up, spontaneous mutations pass undisputed as the driving force of variation and thus speciation. According to this view, in a second step, choice or selection determined by the environmental conditions sees to it that only the most fitting forms survive, thus limiting the variation which arises.

In spite of the many confirmations of the theory based on spontaneous mutation this article aims to outline and provide support for another possible theory, one in which the environmental conditions do not merely select, but direct and bring about variation. This is not intended to cast doubt on the reality of spontaneous or chance mutation, but rather to challenge its claim to absolute and exclusive validity.

The current situation in modern genetics is like that which prevailed in physics at the beginning of the 20th century. Just as at that time wave and particle theories of light were shown to be complementary views, it will be demonstrated that the present theory of chance evolution of organisms must be enlarged to include a complement-ary one, namely directed evolution.   The theory of spontaneous mutation

is placed beside that of adaptive or selection-induced mutation. Which of the two types of genetic change is realised depends on the physiological circumstances and the environmental conditions. These two types of change require different concepts for describing the relation of organism and environment and are dependent upon different molecular processes. Whether complementarity in genetics will have paradigmatic consequences for the overall understanding of living nature or whether, like complementarity in physics, it remains without effect on a wider public, remains to be seen.

There has been no shortage of attempts to develop concepts of variation other than that of spontaneous mutation. The best known goes back to Lamarck[1]. His was the first attempt in a modern scientific approach to evolutionary theory to explain how organismic variety arises. Lamarck's idea of inheritance of acquired characteristics, as discussed in more detail by Lefèvre[2], formed an important though not central support for the theory. Whether it is justified to treat 'inheritance of acquired characteristics' and 'adaptive mutations' as synonymous is discussed in more detail below. Both Darwin[3] and Haeckel[4] embedded the inheritance of acquired characters. Because of this, Haeckel's biogenetic law was largely rejected (c.f. De Beer[5]).

As controversial as adaptive mutation is amongst modern biologists, as certain does its underlying evolutionary principle render service to convinced Darwinists (e.g. Mayr[6]) as an explanation for cultural evolution. Cultural advance is unthinkable without the passing on of acquired characteristics. Experiences are received inwardly and as capabilities are passed on to others (descendants). This principle is essential to the evolution of human communities. If one asks which quality is fruitful for this kind of evolution, the answer has to be co-operation. But the same question posed of Darwinian evolutionary theory gives competition as its answer. The demonstration of adaptive mutations in modern genetics is a contribution to a new understanding of nature. At the same time it leads to a humanising of natural science in that in this kind of genetic change the central human evolutionary principle finds expression in organic nature.

Spontaneous mutations

To understand the discoveries which have led to the concept of adaptive mutations, it is necessary first to be clear about the premises which gave rise to the theory of spontaneous mutation. This also means dealing with molecular interpretations.

Although genetic research was initially confined to plants and animals, bacteria soon played a significant part in answering the questions which arose. Procedures for producing pure cultures of totally different strains as well as for characterising toxin or viral resistance genes were a precondition for genetic experimentation. The other precondition comes from the bacteria themselves. Short generation times and large cell numbers made experiments possible which with other organisms would have lasted years and taken up a vast amount of space. In addition, as bacteria, having only one chromosome, are haploid, genetic changes usually show up phenotypically immediately after they have occurred.

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Despite these advantages interpreting genetic changes proved to be difficult because the results were not reproducible. Whilst it is true that the phenomenon of bacterial virus resistance could be observed on repetition, the number of resistant cells in each replicate experiment exhibited wide variations.

Luria and Delbrück[7], from studies for which they later received a Nobel prize, suspected that it was just this observed variability which might explain how virus resistance comes about in bacteria. They neatly hypothesised that if resistance is *acquired* by contact with the virus the number of resistant bacterial cells should be proportional to the total number of cells used in the experiment, provided that the probability of cells becoming resistant is the same for all cells. A series of identical parallel experiments would thus allow one to expect a Poisson frequency distribution of resistant cells. But if the mutations occur *spontaneously* in bacterial cultures before contact with the virus, then the number of resistant cells should be independent of the total number of cells used in the experiment - provided that the mutation event is very rare - and would simply depend on the time elapsed between the appearance of the mutation and contact with the virus. If the mutation occurs long before virus contact, the number of resistant cells will be large. If it occurs only a short time before contact, the number will be correspondingly small. The frequency distribution of resistant cells from parallel experiments is clonally determined. All resistant cells come from one and the same parent cell. Testing the variance or fluctuation can thus allow a conclusion to be drawn as to the kind of mutation which has arisen.

In their experiments Luria and Delbrück inoculated between ten and twenty tubes containing nutrient broth with 50 to 500 cells of a virus sensitive strain. After a few hours incubation the cell densities rose to about 10^9 cells/ml. 0.1 ml of each liquid culture was spread on petri dishes containing culture medium treated with a large number of bacterial viruses (ca. 10^{14}). After overnight incubation, resistant cells formed colonies visible to the naked eye. Almost all the bacteria plated-out (ca. 10^8) were destroyed (lysed) by the virus and died.

In accordance with expectations, the results were unequivocal. The fluctuation in the number of resistant cells in the cultures tested in parallel was very great. In one experimental series there were petri dishes with no colonies and some with more than 500. The distribution of resistant cells clearly showed itself to be clonal. The mutation event most probably must have arisen before virus contact had taken place and must therefore be spontaneous or 'chance'. The virus simply selected the resistant cells.

This result was in total agreement with the hypotheses of Darwinian evolution. The resulting excitement was so great that Delbrücks warning at a conference in 1947 not to generalise from his discovery went unheard (see Stahl[8]). After the presentation of a paper by Ryan *et al.*[9], in which it is shown how the number of genetic changes in a metabolic mutant increased in a matter of days, he said 'In the case of mutations of bacteria ... to phage resistance ... the phage does not cause the mutations. In your case of mutations permitting the mutants to utilise succinate... as a sole carbon and energy source ... it is an obvious question to ask whether this particular medium had an

influence on the mutation rate.... One should keep in mind the possible occurrence of specifically induced adaptive mutations'.

Another milestone in the development of a theory of spontaneous mutation was reached when Lederberg and Lederberg, using their replica-plating method, managed to isolate from a virus-sensitive bacterial strain cells which were resistant to the virus without having come into contact with it[10]. This showed that resistance mutations arise spontaneously, that is without contact with the selecting agent.

The discovery of the double-helical structure of DNA by Watson and Crick[11] and the biochemical investigations of the replication events in the material of inheritance (c.f. Alberts *et al.*[12]) made possible an explanation of spontaneous mutation. In principle perfect replication of the material of inheritance is guaranteed by the physico-chemical conditions of its molecular structures. A host of proteins participate in this synthesis and minimise the errors which arise during replication. Such errors can manifest as mutations and are interpreted as the reason why evolution happens at all. It is also clear that the faithfulness of replication of DNA is directly proportional to the size of the genome (the quantity of the substance of inheritance) (c.f. Maynard Smith[13]). The smaller the genome the higher the mutation rate. Put another way, a text with thousands of words can be transcribed many times without distorting the meaning when one wrong word is substituted in every ten thousand. If errors were to occur with the same frequency in a text with a hundred thousand words, ten words would be altered at each transcription. With frequent transcription, distortion of the meaning could not be ruled out. To avoid unacceptable changes, the transcription accuracy would have to be increased.

If during replication of the material of inheritance the mutation rate is too high it could have catastrophic consequences for the organism concerned. But if DNA replication were absolutely perfect, undirected 'chance' evolution of living organisms would be rendered impossible. Spontaneous mutations are an essential component or instrument of the evolution of all living beings on earth. Such mutations are not determined by environmental conditions but arise mainly through replication of the material of inheritance.

Adaptive mutations: the concept clarified

In order to deal properly with adaptive mutations it is necessary first to clarify a misunderstanding and a conceptual confusion. Equating the concepts 'inheritance of acquired characteristics' and 'adaptive mutations' is often criticised (c.f. Lenski *et al.*[14]). The first explicitly emphasises the fact that characteristics must first be formed before they can be passed on. But the second concept implies that known mutations are seen to revert to the wild type. After a reversion event the cells concerned exhibit characteristics that were shown by their ancestors prior to the mutation. Reversions provide modern genetics with a tool that allows phenotype and genotype to be kept equally in view. I will use the two expressions 'inheritance of acquired characteristics' and 'adaptive mutations' synonymously, because in both cases it is true to say that there must be an effect on the material of inheritance directed from the environment

understanding. Bockemühl's work on groundsel (*Senecio vulgaris*) provides striking examples of such an understanding[52].

Spontaneous mutations also have a part to play in a monist evolutionary theory. They show that genetic changes can also occur in the relation of the organism to itself. They are complementary to adaptive mutations. The polarity of world relation and self relation and its overcoming through the organism itself are hallmarks of the living. I have attempted to elaborate this in studies on developmental processes in amphibians[53]. Similar polarities have come to light in other studies (Pankow *et al.*[54], Schad[55], Suchantke[56,57]). Spontaneous and adaptive mutations are not causes of the variation in form and function, but results of a variety of organic processes. These thoughts will be extended in a further article. Furthermore, I shall report on experiments investigating the existence of adaptive mutations in *Drosophila*.

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Piaget[23], the Swiss developmental psychologist, provides a third fundamental consideration. The idea of adaptive mutations was a logical outcome of his investigations into cognitive processes and their biological basis. It is an undoubted fact that the human being gains knowledge by constantly taking in experiences and as a result of this process is able to pass on faculties. According to Piaget the act of cognition is only possible through (subjective) receptivity and (objective) external stimulus. It produces a relationship between object and subject. Thoughts are contoured by percepts and determine our intentions. Intentionality fixes what is extracted from the sense perceptible world and determines the framework of elements of observation. As zoologist, I look primarily at animals and through an interest in morphology further restrict observations to their form.

Both aspects, thinking and perceiving, reciprocally determine and alter one another. Because cognitive processes have and must have a biological basis, this in turn must have a functional structure like the cognitive process itself. Organic regulatory processes follow the same laws as those of cognition: they undergo adaptive change and development, not only physiologically but also genetically. Piaget's morphological investigations of the water snail (*Lymnaea stagnalis L.*) in a wide range of habitats appeared to confirm the hypothesis of adaptive mutation.

The discussion of these three authors provides more than a foundation for a theory of adaptive mutation. All three overcome the materialistic tendency in the modern view of heredity. With Steiner, the overcoming is quite explicit. Development and heredity can only be grasped through a combination of sensory and supersensible processes. All living organisms are an expression and result of ideal-material processes. The unity of matter and spirit is the basis of Steiner's monist evolutionary theory.

With Waddington the overthrow of the materialistic view of heredity is reflected in the idea of organismic wholeness, which is not to be thought of as solely material. Relationships and interactions with the environment belong just as much to the organism as to its organs, cells and molecules. The basis of his evolutionary theory is unified life of organism and surroundings.

Finally, Piaget postulates unity of cognitive and living processes. As psychologist he did not doubt the former, nor as biologist the latter. One could describe his theory of development as a monism of the soul, keeping consciousness and body together.

Complementary genetics and enlivening of the concept of heredity

To quote Steiner[50]: 'the essence of monism is the idea that all occurrences in the world, from the simplest mechanical ones to the highest human intellectual creations, evolve themselves naturally in the same sense, and that everything which is required for the explanation of appearances, must be sought *within* that same world.' In relation to adaptive mutation, this view means, as we have seen, that genetic changes must be understood as an expression of an interrelationship between living organism and its habitat. Steiner, Waddington and Piaget have shown approaches to such an

and the living organism. Furthermore, new characteristics that must be inherited can manifest only through modification of already existing heritable material.

Another difficulty concerns the view that the theory of adaptive mutations is 'Lamarckian' (c.f. Marx[15], Symonds[16], Mayr[6]). There are several objections to this. As already mentioned Darwin and Haeckel include the inheritance of acquired characteristics in their theories, although they have both expressly countered Lamarck's teleological evolutionary theory. The term 'adaptive mutations' expresses the fact that the constraints of life and the environmental conditions not only work selectively on preformed characteristics, but also can determine new ones. Such characteristics can be described as 'goal-directed' without, like Lamarck, presupposing an evolutionary goal. Even Darwin[3] coined an expression for this: 'Effects of habit and the use or disuse of parts'.

Early supporters of the theory of adaptive mutations

Since Mendel, adaptive mutations became a topic of increasing interest and was described in reputable journals. One of the most outspoken representatives of the theory was Kammerer. On one of his trips to the USA he was even heralded by the newspapers as the 'new Darwin' (Koestler[18]). In many publications and using a wide variety of animals he sought to demonstrate the existence of the inheritance of acquired characteristics (c.f. Kammerer[19,20]). He described them for the midwife toad *Alytes obstetricans*. By raising the temperature of its surroundings the animal can be made to depart from its usual behaviour of reproducing in water. Under the new conditions the male forms 'nuptial pads'. These thumb-like structures occur on the forelimbs of many amphibians that reproduce under water. It is thought that they help the males get a better hold during copulation. After copulation on land, the male carries the strands of spawn containing the fertilised eggs around with him wrapped round the hind leg until the larvae hatch. Under the new conditions the spawn remains in water. The tadpoles which have undergone their embryonic development in water exhibit external gills similar to the larvae of other toads and frogs.

Both nuptial pads and external gills can be regarded as an expression of an adaptation to the new conditions. Both features also appear in subsequent generations even when the animals are returned to normal living conditions. They appear to be genetically fixed.

More convincing were the experiments with the sea squirt (ascidian), *Ciona intestinalis*. Kammerer described them as providing the most significant evidence for adaptive mutations. After repeated amputation of the terminal tubes which are used for feeding and excretion, these organs grow extremely long. Specimens with long tubes give rise to long-tubed offspring thus giving rise to the supposition that inheritance of acquired characteristics is involved. To exclude the possibility of prior chance mutation causing the long tubes, Kammerer removed the gonads. After regeneration of these organs long-tubed specimens once again developed out of the newly formed germ cells. Thus it seemed that clear evidence for acquired characteristics had been obtained.

Kammerer's experiments are clearly described and from their methodical structure withstand critical appraisal today. Nevertheless alternative explanations such as cytoplasmic or maternal effects that could bring about developmental modifications without changing the DNA would nowadays have to be excluded. In view of the tragic circumstances of Kammerer's death, which is interpreted as admission of his scientific fraud, Koestler[18] emphasised the need for a repeat of these experiments.

In Russia, Mitschurin (see Sankjewitsch[21]), using the most varied cultivars investigated the questions of environmental influence on seeds and rootstock on fruit. He too observed environmental influences which were genetically fixed. But his work fell into disrepute and oblivion probably through the political polemic from and surrounding Lysenko and his unsuccessful wheat verbalisation experiments.

Waddington[22] and Piaget[23] reported theoretical considerations, suggestions and descriptions regarding experiments on adaptive mutations which will be discussed below. At the level of molecular genetics, the phenomenon of adaptive mutations has been reported for flax (Marx[15], Cullis[24]).

Adaptive mutations since 1988

The publication of evidence for adaptive mutations by Cairns *et al.*[25] brought about a change. The standing of both the author, as former director of the respected Cold Spring Harbour Laboratory, together with that of the journal *Nature* in which the work was published, left little doubt as to the scientific quality of the work and sparked-off discussion and controversy which has lasted to this day. Many 'main stream' geneticists felt obliged to take positions and carry out further experiments. Since then there have been a considerable number of publications describing adaptive mutations for various micro-organisms and cellular anabolic and catabolic processes. Furthermore some authors tend to the view that this form of inheritance also plays a part in tumour formation (for reviews see Foster[26,27]).

Cairns' group investigated the frequency of reversion of a well known and genetically characterised metabolic mutant *lac* in *E.coli*. Cells with this mutation can no longer use lactose and are dependent for their growth on glucose or another sugar in the growth medium. The reversion of the mutation to lac^+ can easily be demonstrated by plating out the cells onto a medium containing lactose and a colour indicator. Revertant cells form red colonies.

In an experimental design based on that of Luria and Delbrück[7], analysis of the frequency distribution of sixty cultures prepared in parallel showed that spontaneous reversions must have taken place before selection. However, others appeared to have occurred adaptively only after contact with lactose the selecting agent. Further observations showed that the number of reversions increased when the petri dishes containing lac^- bacteria were incubated for several more days. Obviously in the course of time more revertants were generated. Control experiments showed that reversions only occurred when the growth medium contained lactose. If this sugar was missing, or only sprayed on the bacteria after one or more days, the number of lac^+ colonies remained unchanged with longer incubation. Finally, it was shown that

They are of course dependent on molecular processes, but they are not determined by them. Thus molecular genetics points to the necessity of looking beyond its current paradigm for alternative concepts and approaches to organisms and their inheritance. Paradoxically this leads first to the rediscovery of theoretical foundations which have been forgotten. I shall illustrate this with reference to the work of three individuals.

I turn first to a pamphlet essay by Steiner[50]. In it the 'inheritance of acquired characteristics' is seen as a consequence of Haeckel's biogenetic law. Steiner emphasises that without this law a monist evolutionary theory has no validity. The essay is in essence against the last vestiges of vitalism and the preformation theory associated with Weismann.

Monistic evolutionary theory signified a big challenge to understand 'being' and 'appearance', requiring one to grasp the organic as a process which takes place as much from top downwards (from idea to world of the senses) as from bottom upwards. In this process both aspects - the ideal in the type at work in the organism and the real as its appearance in the world of the senses - undergo changes and metamorphoses in reciprocal interdependence.

These ideas are substantially developed in an earlier essay by Steiner[51] in which the relationship of the Goethean idea of type (archetype) to the organism which actually manifests is clarified and discussed. It is the essence of all living organisms that they respond inwardly to the experiences they undergo in their development and thus eventually pass them on to their offspring.

Waddington[22] offered a further theoretical principle. According to him there are two distinct possibilities for genetic variation. One concerns isolated features which are altered accidentally. Industrial melanism in the peppered moth *Biston betularia* is a textbook example of this and for modern evolutionary biology provides irrefutable evidence for the occurrence of spontaneous mutation. The other possibility for genetic variation concerns features which are embedded in the totality of the organism and its environment. Waddington's example for this is the forequarters of the gibbon and pangolin (scaly ant-eater): the gibbon's forelimbs point in their slenderness, length and exceptional mobility to activities such as climbing and hanging, whereas the form of forelimbs of the scaly ant eater exhibit rigidity, shortness and compactness of bone formation which are easily comprehensible in terms of a digging function. Both animals reproduce in their entire bodily make up the specific orientation of their different activities and modes of behaviour. The logical construction of the entire form is unmistakable. According to Waddington it is extremely unlikely that the extremities have come about by a large number of accidental individual changes. It seems more plausible that they were formed through the specific behaviour of the animal in its respective habitat during the course of the evolution of the species.

Waddington used *Drosophila* experiments to develop the concept of organismic totality and adaptive reaction to specific qualities in the surroundings. He described short term physiological changes that had become genetically fixed 'genetic assimilation'. Such changes as take place over a long period, he described as 'evolutionary adaptation'.

reversion rates during exponential growth with those during the stationary phase. The rates were as much distinguished by the two culture conditions as by the individual strains. Base pair substitutions, insertions and deletions are dependent on the physiological state of cells and the type of change in the environment.

Unlike spontaneous mutations, adaptive mutations are dependent upon various components of the recombination system (RecA, RecBCD, Harris et al.[45]). Under normal conditions, this system mediates homologous recombination between chromosomes and enables insertions and deletions of DNA sequences in the bacterial chromosome. If the proteins of the RecBCD system are lacking, adaptive mutations no longer take place. These findings have been described as progress towards the understanding of genetic intelligence (Thaler[46]).

Another piece in the jigsaw was the discovery that not only the recombination system but also intercellular DNA transduction, the transfer of genetic material during bacterial conjugation (a kind of primitive sexual pairing), participates in the appearance of adaptive mutations.

Conjugation proved significant is several ways: for bacterial strains which had the selective gene on the chromosome rather than on the transduction plasmid, the reversion rate was 25 to 50 times smaller (Radicella et al.[47], Galatski & Roth[48]). Removal of the conjugation apparatus with detergents or additional mutations in the enzymes of the transfer function reduced the adaptive mutation rate to the same extent.

According to Shapiro[49], these results have far reaching consequences for evolutionary theory, although he also holds that they make the hypothesis of 'directed mutation' superfluous. The transfer of the transduction plasmid is dependent on DNA replication, which is why mutations associated with chromosomal replication can occur by 'chance'. Even so the results show that the rate of meaningful mutations can be significantly increased by selection and that by transduction, which can be regarded as a primitive form of intercellular communication, meaningful mutations can be passed on. Recombination and plasmid transfer are cellular functions which allow an active reaction to its environmental conditions. A significant component of genetic variation is without doubt no longer attributable solely to chance events in the replication of the material of inheritance, but can only be understood by considering the relationships between living organism and the world in which it lives.

Non-molecular concepts of adaptive mutation

I hope to have shown in the foregoing that modern genetics has reached a turning point. But true insight as to the significance of adaptive mutations cannot be gained through describing molecular processes. This is because, by reducing the phenomena to molecular processes, the fundamental and qualitative differences between spontaneous and adaptive forms of inheritance are overlooked. The description gets lost in detailing DNA-protein interactions. But the differences lie in the possibility of manifesting in the most varied of ways relationships and interconnections between organism and environment and of making these available to the next generation.

with mutations such as val^R, which are not selectable, no reversions occurred. Increase in the reversion rate only resulted when it was 'useful' for the multiplication and growth of bacteria. They were without doubt adaptive, or, as Cairns' group put it, directed.

The results stood in contradiction to the theory of spontaneous mutations. The reversions occurred only during selection and in appropriate environmental conditions. Lactose had to be present. The medium appeared to 'entice' out the reversions. Particularly noteworthy is the fact that they only took place during the stationary phase when DNA replication errors cannot occur. None of these observations were new. In 1961 Ryan's group had already published work suggesting mutation events without replication (Ryan et al.[9] and Symonds[28]), but this received little attention amongst geneticists. The Cairns' group managed only to publish once more in their entirety the most important observations evidencing non-spontaneous mutations.

The Cairns work is also noteworthy for another reason. Since 1943 bacterial genetics has concerned itself with cells in the exponential growth phase and investigated many phenomena which determine the life and death of bacteria. But adaptive mutations occur only when cells are not dividing and even then only when the genetic change is choosing between growth/division or rest. For this reason it is possible to speculate as to the significance of adaptive mutations for natural conditions. From the still young science of the genetics of the stationary phase, there are reports which suggest that adaptive mutations occur also under natural conditions (Kolter[29]).

Hall's work

Adaptive mutation research was greatly extended in variety and scale by Hall, a microbiologist based in Rochester (USA). Working intensively with the conceptual problems of the new theory, he investigated several organisms and catabolic processes as part of his interest in reversions of point mutations (substitution of individual base pairs) and deletions. The conclusions he drew from this were uncertain and provisional. Where observed changes were at first adaptive (Hall[30]), they later were explained as spontaneous (Hall[31,32]), or occasionally in the following paradoxical way 'Spontaneous point mutations that occur more often when advantageous than when neutral' (Hall[31]). These he called 'selection induced mutations' in a later publication (Hall[33]), and he ultimately reached the conclusion that there is indeed a phenomenon of adaptive mutation, but there is no explanation for it (Hall[34]).

Hall's initial work was on the double mutation in the bgl operon in E.coli (Hall[30]). This operon codes for the necessary enzymes for the catabolism of glucosides. The individual reversion rates experimentally determined for the two mutants is 4×10^{-8} and $< 2 \times 10^{-12}$ per cell division. Assuming that the two mutations are independent from one another, in bacterial strains with both mutations the reversion rate, given by the product of the two individual reversion rates, is 8×10^{-20}. Such an event would

never be observable under experimental conditions because at least 8×10^{20} cells would need testing, thus requiring at least 100,000 litres of liquid culture. Bacteria incubated for two to three weeks in petri dishes formed colonies of revertant cells able to catabolize glucosides. The reversion rate of was 2×10^{-8}, far higher than expected. Here too reversion managed to take place in the stationary phase and only when glucosides were present in the medium.

Further work investigated point mutation behaviour in the tryptophan operon in *E.coli* (Hall[31,33,35]). Once again the reversion rate was far higher than was expected on the basis of spontaneous mutation and appeared under conditions of selection. The author also demonstrated that reversion was independent of DNA replication and increased according to the length of time cells were in contact with the selective substrate. Control experiments ruled out the possibility that cryptic growth of cells or retarded division of pre-existing revertants determined the reversions. Experiments with baker's yeast *Saccharomyces cerevisiae* (Hall[36]) showed that adaptive mutations can also be demonstrated for eukaryotes.

Objections and attempts at a molecular explanation

Critical and partially justified objections to the idea of the existence of adaptive mutations were not slow in appearing. Several experiments were repeated with more stringent controls. The mobilisation of the bacterial virus *Mu* which the Cairns group[25] observed and interpreted as a directed mutation proved to be a spontaneous mutation (Mittler and Lenski[37]). The high reversion rate which Hall[30] had observed with double mutants was explicable in terms of the growth of intermediary genotypes (Mittler and Lenski[38]). Finally it was shown that the difference in reversion rates between two independent mutations (Cairns *et al.*[25]) could be ascribed to known physiological processes (MacPhee[39]).

The criticism had the result that in subsequent work the necessary control experiments were carried out. Thus in his investigation of the reversion of mutations in the tryptophan operon Hall[34,35,36] was able to rule out that adaptive effects were arising through intermediary growth or death of cells. Both possibilities would have given a deceptive nominal increase in the mutation rate thus allowing spontaneous mutations to appear as adaptive events (Mittler & Lenski[40]). The criticism as to the reality of adaptive mutations eventually led to their experimentally verified acceptance.

Still unsolved was the question of how adaptive mutations could occur. The search for an explanation based on the underlying molecular processes was linked to the hope that phenomena which would not fit in could nevertheless eventually be interpreted 'classically'. The lynch pin in the structure of modern genetics is still its central dogma which states that 'information' flows only from the material of inheritance to the protein (DNA>RNA>protein). This underpins the idea that heritable changes are never determined by protein. The phenotype has no influence on the genotype. Since the discovery of retroviruses, whose viral RNA chromosome after successful infection is transcribed into DNA, the dogma is only partly valid. Adaptive mutations now threaten to overthrow it completely.

To explain adaptive mutations, various working hypotheses were formulated (summarised in Koch[41]) which, under selective conditions and with known molecular mechanisms would have allowed a raised mutation rate to be assumed. The postulates of three most important hypotheses are stated here.

Hypermutability: The basic mutation rate in bacteria under stress conditions is significantly raised (Symonds[42], Hall[31]) and that amongst many chance mutations some also occur which are selected.

Increase in the mutation rate through reverse transcription (Stahl[43]): In cells in the stationary phase there are always transcription processes going on, i.e. DNA is transcribed to RNA. It is known that in these processes the transcription accuracy is relatively small and thus the mutation rate is increased. RNA molecules arising in this way which enable the synthesis of a protein necessary for growth can, after being changed to DNA, replace the original chromosomal sequences.

Slow repair (Stahl[43]): Under stationary phase conditions small pieces of DNA are broken down and resynthesised. The repair mechanisms which normally replace wrongly inserted nucleotides are not active.

All hypotheses were experimentally tested and had to be rejected. With hypermutability the frequency of the adaptive reversions in the *trp* operon signified a mutation rate of 0.04 per base pair (Hall[32]). Thus on average every 25th base pair would have to be substituted. Such a high rate would without doubt have been lethal for the bacteria. Hall investigated the relevant gene locus by sequencing to determine whether, in the neighbourhood of the necessary reversion, other substitutions had taken place. But he was without success. Such a 'directed' localised increase in the mutation rate would however have only postponed the crisis of finding an explanation.

The second hypothesis also had to be rejected (Hall[31]) because with some bacterial strains which exhibit adaptive mutations no reverse transcriptase activity has so far been demonstrated.

The slow repair hypothesis failed because as well as the expected selective mutations, independent mutation events in other genes would also have had to occur (Hall[32]). In no case could these be detected.

That the molecular basis of adaptive mutations is of a non-classical kind was revealed by a series of unexpected results, which, however, in retrospect an unprejudiced observer would hardly wonder at. Adaptive mutations always appear in the bacterial stationary phase. DNA turnover is minimal. Mutation events are time dependent. But spontaneous mutations occur by maximal DNA turnover in the phase of exponential growth and are dependent on replication.

A first indication of the difference at the molecular level in the occurrence of the two types of mutation was given by the analysis of the spectrum of reversions under selective (adaptive) and non-selective (spontaneous) conditions (Hall[44]). Thirteen strains with different mutations in the same gene (*lac*) were used to compare