Science and Mathematics Group of the Anthroposophical Society in Great Britain Newsletter – Autumn/Winter 2020

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Articles

Kepler's Search for the Creative Harmonies

"*Not every intuition is false. For Man is the image of God, and it is quite possible that in his exploration of the world's adornments, his intentions are the same as God's. For the world partakes in quantities, and the human spirit comprehends nothing as clearly as quantity, for the understanding of which he has evidently been created*."

Kepler in a letter to Chancellor Herwart von Hohenburg.

"*The ideas of quantities have been and are in God from eternity, they are God himself; they are therefore also present as archetypes in all minds created in God's likeness. On this point both the pagan philosophers and the teachers of the Church agree*."

Kepler in his introduction to *Mysterium Cosmographicum*.

Overture

Johannes Kepler (1571-1630) has gone down in history as a great mathematician and astronomer, renowned for his discovery of the three laws of planetary motion named after him. He lived in an age when empirical science and the use of mathematics was beginning to overtake theoretical speculation, and observations which contradicted the opinions of ancient philosophers such as Aristotle were no longer rejected out of hand. Although a new age was dawning, Kepler nurtured a deep and lifelong interest in an ancient cosmology, which he sought to revitalise by making use of his considerable mathematical prowess. He explained his ideas in several highly original books, in particular *Mysterium Cosmographicum* (published in 1597) and *Harmonice Mundi* (published in 1619), which have mostly been ignored by scientists, even though the latter contained his third law, arguably the most important law in astronomy discovered in the 17th Century. Kepler repeatedly stressed the importance of both these works, and he considered *Harmonice Mundi* to be the fulfilment of his life's work.

Kepler was well aware of the major changes taking place in human consciousness set in motion by Copernicus and the Renaissance. He considered it his calling to develop a cosmology which united recent discoveries in astronomy, several of which had been developed by him, with the essence of ancient traditions. His search for the foundations of cosmic harmony was central to this work. By taking this ancient idea, by attempting unsuccessfully to integrate it into the birth of modern astronomy, and by wrestling with the inconsistencies of his failed attempts, he discovered the three laws of planetary motion, still valid today.

Kepler had intended to become a Lutheran minister, and spent five years at the seminary in Tübingen (Germany). Here he was introduced to the ideas of Copernicus by his mathematics teacher Michael Mästlin, and he soon became a strong defender of a heliocentric universe. His growing interest in astronomy caused him to abandon his vocation when he realised that he could still contemplate God by studying the heavens. He believed that the harmony and beauty inherent in the universe were the means towards an understanding of God: 'I was determined to be a theologian; I was distressed by this for a long time. But look! Even in astronomy my work worships God.' In 1594 he left the seminary to take up a teaching post in Graz (Austria).

We know from his correspondence with Herwart von Hohenburg [1] that Kepler made use of the monochord (an instrument with only one string and a moveable bridge) when he began to study musical harmony in 1599. He wanted to discover the consonant intervals by using his sense of hearing. (Kepler had poor eyesight – not a good precondition for an astronomer!) His intention at the time was to write a cosmology based on musical intervals – an intention which had to wait twenty years for its fulfilment. In the course of his musical studies he rediscovered the natural harmonics first explored by Pythagoras two thousand years earlier. Like Pythagoras, Kepler believed that because God had created humankind in His image, it would be possible to find the harmonies underlying all of creation by intent listening.

Pythagoras

The harmony of the spheres is usually considered a fanciful myth going back to Greek times, and in particular to the philosopher Pythagoras (6th Century BCE). Pythagoras and his followers believed in the unity of the cosmos; the observations of astronomy, the cycles of time, the intervals of music, the ratios of arithmetic, and the concepts of geometry were all seen as grounded in a harmonious and overarching universal structure; moreover, this same harmony was inherent in the structure of the human mind.

The Greeks had two entirely different words for number, one expressing quality (*arithmos*), the other quantity or size (*megethos*). The supreme importance given to the study of number as quality was based on a realisation that only a deep understanding of numbers and their relationships enabled the human mind to recognise the universal harmony.

Pythagoras studied geometry with the Ionian philosopher Thales, and arithmetic and music with the priests of Thebes in Egypt. In Egypt all education was controlled by the temple priests. This was to ensure that those aspiring to leadership studied and worked according to the sacred canon of number and proportion, for only in this way could the ancient wisdom be preserved. The purpose of Pythagoras and his school was to keep these traditions and the inspirations associated with them alive.

The main musical instrument in use at the time was the Egyptian shoulder harp, which like the modern harp, had strings of different lengths. Pythagoras is credited with the discovery that the sounds produced by the strings depended on their lengths, that these lengths could be described by numbers, and that the ratios [2] of these numbers (the relative proportions of the length of the strings) determined whether the sound was harmonious or not.

Legend has it that as he was walking past a blacksmith shop, he heard the sounds of many different hammers beating the anvils. He stopped to listen, for the hammers created a harmonious sound. But there was one exception, which created a distinctly unpleasant sound to his ear. He ran into the shop to investigate, and discovered that, with one exception, the weights of all the hammers were in simple proportions to each other. In other words, hammers with half, two thirds, or three quarters of the weight of a particular hammer, all generated harmonious sounds. On the other hand, the hammer that was generating disharmony when struck along with any of the others had a weight that bore no simple relationship to the other weights.

Pythagoras was inspired to carry out further investigations with different materials, and he applied his discovery to a variety of sounds. He used bells, different volumes of liquid in equal sized glasses, and different tensions in strings of equal length, adjusted by suspended weights. Figure 1 shows a 15th Century woodcut illustrating Pythagoras' musical investigations.

Tubal is Tubal Cain, a biblical figure known for being the first metal worker and blacksmith (Genesis 4:22). Philolaus (c. 470 – c. 385 BCE) was a Greek philosopher and follower of Pythagoras. He believed that the foundation of everything is the harmonious combination of the finite and the infinite [3].

Through such investigations Pythagoras put on a firm foundation what ancient peoples had long intuited. Two or more tones based on small whole number (integer) relationships produce harmonious sounds that are easy on the ear and pleasant to the soul. The weight of a hammer, or the length of a string, could be described by numbers, and the sounds produced by these 'instruments' were perceived as harmonious when the numbers were in in simple relationships with each other.

Pythagoras thus demonstrated that musical intervals, i.e. the difference in pitch between two musical tones, could be explained by numerical ratios. This is generally recognised as the first rational attempt to explain human experience objectively, thereby providing a bridge between quantity and quality, between the human and the-

Figure 1 from 'Theorica Musicae' by Franchinus Gaffurius

divine

The ancient Greek word ἁρμονία (*harmonia*) did not denote harmony as understood today, but meant rather 'union', 'agreement', or 'concordance' with the underlying order of the universe. Music, astronomy, arithmetic and geometry were always studied together (the quadrivium), a practice which continued right into the Middle Ages. Figure 2 shows Musica and Pythagoras on the royal portal of Chartres cathedral.

Most famously, Pythagoras experimented on a monochord. By stopping the string at various lengths, he discovered the harmonic overtones [4]. Together with the open string, its different lengths vibrate two, three, etc times as fast. There is an inverse relation between the string length and the vibration frequency. Half the length, produces twice the frequency, and a pitch double that of the open string. Stopping a string at two-thirds (2:3) its length (from thee right) produces a pitch one and one-half (3:2) times that of the open string, an interval known as a perfect fifth. See Figure 3.

Figure 2

The teachings of Pythagoras were handed down orally for several centuries, and were first written down in the days of Aristotle (384-322 BCE). Since that time, they have formed an essential chapter in the study of music through the ages.

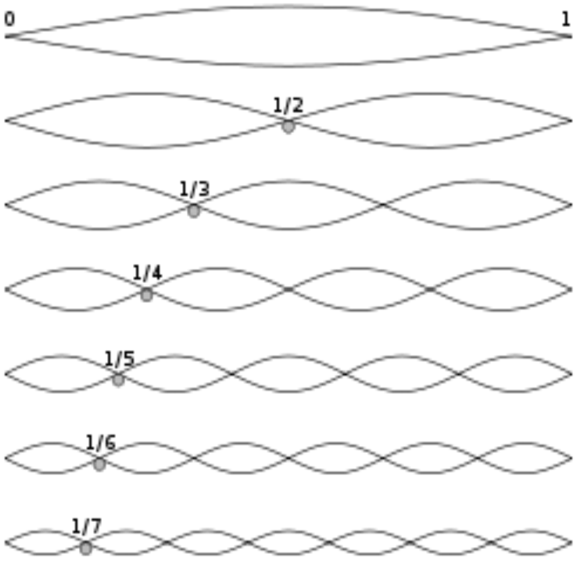


Figure 3 The vibrations of a plucked string when stopped at different lengths. Each vibration sounds its own harmonic.

Figure 4 shows the basic Pythagorean intervals in a detail from Raphael's fresco in the Vatican usually known as 'The School of Athens' [5]. It shows the Pythagorean harmonic intervals; tone (*epogdoon*) [6], fourth (*diatessaron*), fifth (*diapente*), and octave (*diapason*). The diagram illustrates the principles of a Greek lyre. The four strings are of equal thickness and under equal tension, with relative lengths 6 (VI), 8 (VIII), 9 (IX) and 12 (XII) units. When plucked,

- the interval between VI and XII is an octave (12:6 = 2:1),
- the interval between VI and IX, and between VIII and XII is a fifth (9:6 = 3:2),
- the interval between VI and VIII, and between IX and XII is a fourth (8:6 = 4:3),
- the interval between VIII and IX is the difference between a fourth and a fifth, which we call a major tone (9:8) today. This is not a harmonious interval [7].

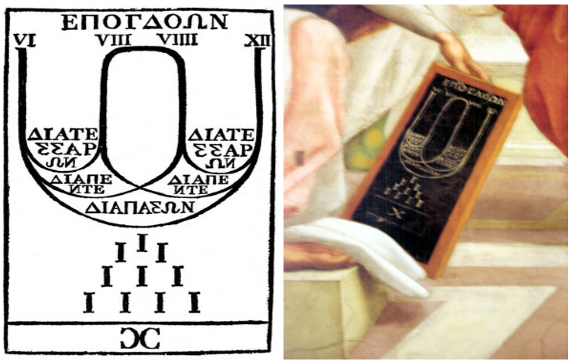


Figure 4 Image from http://arthistoryresources.net/renaissance-art-theory-2014/pythagoras-music-proportion.html

Under Raphael's diagram is a triangular figure composed of four rows of ones (I). This is the tetractys, which is the Pythagorean perfect number 10, the sum of the first four numbers (1, 2, 3, 4), which make up the intervals played on a Greek lyre. These harmonies were used to accompany speech, but not song. Moreover, the Pythagorean number 10 comprises all numbers, and thus was regarded as sacred and as the "mother of the universe."

The Spheres

'*How sweet the moonlight sleeps upon this bank!*
Here will we sit and let the sounds of music
Creep in our ears: soft stillness and the night
Become the touches of sweet harmony.
Sit, Jessica. Look how the floor of heaven
Is thick inlaid with patines of bright gold:
There's not the smallest orb which thou behold'st
But in his motion like an angel sings,
Still quiring to the young-eyed cherubins;
Such harmony is in immortal souls;
But whilst this muddy vesture of decay
Doth grossly close it in, we cannot hear it.'

Lorenzo in Act V, scene 1 in *The Merchant of Venice*

The idea of heavenly orbs, or spheres, carrying the planets on their celestial journeys, also goes back to Greek antiquity. It was first recorded in the cosmology of Anaximander (a contemporary of Pythagoras) in the 6th Century BCE. Plato (428-348 BCE) believed that the universe was a perfect creation, and that therefore the orbs had to be spherical, with a crystal sphere supporting the fixed stars. In the *Timaeus* [8]. he describes the construction of the universe (*kosmos*) according to ratios corresponding to musical intervals, thereby offering an explanation of its order and beauty. His pupil Eudoxus developed the idea of concentric spheres supporting the planets, with three or four spheres needed to support each of the seven planets [9]. Aristotle increased the number of spheres to 47, and explained that the gods were able to bring about their rotation about their common centre Earth, by being loved [10]. He wrote that Pythagoras and his followers believed that the relative motion of the spheres produced musical tones. Nested as they were within each other, a kind of ether friction created harmonious sounds because the distances between them were believed to be in the same proportions (ratios) as the proportions of the tones in consonant music.

He explained that the reason no one was able to hear these harmonies (except possibly Pythagoras himself) was because they are with us since birth, and never having experienced their absence, we are unable to notice their presence. Aristotle himself was unable to accept any of this, and dismisses the celestial harmony on two counts: first, no one is able to actually hear the music, and second, given that all physical objects produce a sound of some kind, with small objects producing soft sounds, and larger objects louder sounds, the enormity of the spheres would imply that the sound they produced would have 'an intensity many times that of thunder' [11].

The Egyptian Claudius Ptolemaeus (about 100-170 CE), better known today as Ptolemy, was one of the dominant figures of ancient science. Although best known as an astronomer, he wrote definitive textbooks on geography and optics. His book '*Harmonics*' discusses pitches, intervals and modulation, concepts which he related to human souls as well as to celestial bodies.

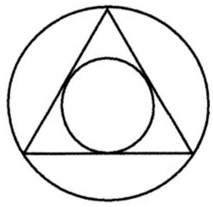
He developed a geometric model of the solar system, with a reduced number of spheres per planet. Although the necessary calculations were complicated [12], Ptolemy's model was able to predict the positions of the planets relative to the fixed stars to a higher degree of accuracy than any of the preceding models. It stood the test of time, and was in use for almost 1500 years, until astronomers realized that their increasingly accurate observations no longer matched the predictions of Ptolemy's model.

The time had come for Copernicus (1473-1543) to develop his heliocentric model, which he did in his consciousness-changing book *De revolutionibus orbium coelestium* (*On the Revolutions of the Celestial Spheres*) published in 1543. Although Copernicus does not discuss the actual nature of the spheres in detail, his few allusions suggest that he did not think of them as physical entities. He placed the sphere of the Moon around the Earth and moved the Sun from its sphere to the centre of the universe. The planetary spheres circled the Sun in the order Mercury, Venus, the great sphere containing the Earth and the sphere of the Moon, then the spheres of Mars, Jupiter, and Saturn. He considered the outermost sphere, the celestial sphere of the stars, to be fixed and unmoving. Although Copernicus' heliocentric model considerably simplified the calculation of planetary positions, it had the disadvantage of contradicting the appearances. His model could be thought, but not experienced directly.

But it could be used to calculate the relative distances of the planets from the Sun, and from each other. When first computed, these distances appeared arbitrary, and their proportions bore no relation to the perfection of a universe created by God. Tycho Brahe (1546-1601), for example, famed for the accuracy of his observations, found it difficult to reconcile the vastness of the calculated distance between Saturn and the fixed stars.

Mysterium Cosmographicum

Kepler published his first book, the *Mysterium Cosmographicum* (usually translated as *The Cosmic Mystery*) in 1597 [13]. On July 9 1595 (he recorded in his diary) in the middle of an astronomy lesson introducing the conjunction of Jupiter and Saturn, Kepler was struck by an idea with such force that it became his *leitmotiv* for the rest of his life. The essence of his idea was that the structure of the universe is built on a small number of regular polygons [14]. He had drawn a diagram on the board, when he suddenly realised that the two circles (representing the orbits of Jupiter and Saturn) drawn inside and outside the triangle (representing the positions of their conjunctions) were in the same proportion as the actual orbits of Jupiter and Saturn.



If an equilateral triangle (the first regular polygon) could be fitted between the orbits of Saturn and Jupiter (the two outermost planets), then, he reasoned, a square should fit between the orbits of Jupiter and Mars, and a pentagon between Mars and Earth, followed by a hexagon between Earth and Venus, and a heptagon between Venus and Mercury.

Although he soon discovered that this two-dimensional model didn't work, he returned to the idea of regular polygons underlying the structure of the universe 24 years later in *Harmonice Mundi*.

Kepler realized that because the planets move in three-dimensional space: 'One has to look for three dimensional shapes, and behold, dear reader, now you have my discovery in your hands.' He had found the three dimensional shapes he needed in Plato's *Timaeus*.

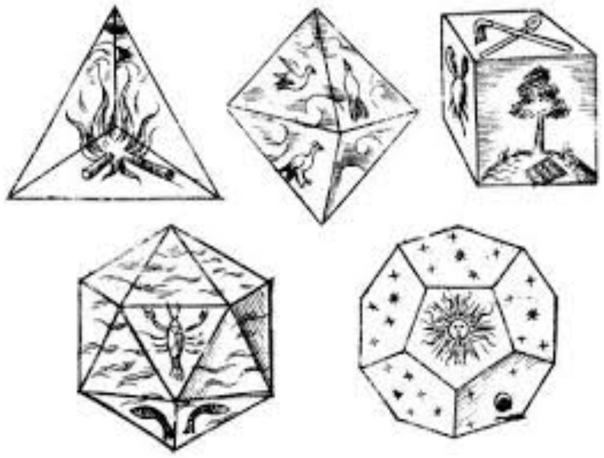


Figure 5 The five Platonic solids as illustrated in Kepler's *Harmonice Mundi*. Their relation to the four (Greek) elements, and the fifth (the quintessence) are described in Plato's *Timaeus*.

By an amazing stroke of coincidence (or luck?) only six planets were known at the time, and there are just five perfect three dimensional solids (polyhedra). These are the Platonic solids. They are perfect because when placed inside a sphere all their vertices (corners) touch the inner surface of the sphere, and when a sphere is placed inside a Platonic solid, the outer surface of the sphere touches every face exactly in its centre. For Kepler, the fact that the five Platonic solids could be perfectly placed within the six planetary orbits (he hadn't yet established that the orbits are in fact elliptical) was proof enough that the universe was created by divine arrangement.

And it worked, sort of. A cube fitted into Saturn's orbital sphere, into which could be fitted Jupiter's orbital sphere, into which fitted a tetrahedron, which held the orbital sphere of Mars. Between the orbital spheres of Mars and Earth came the dodecahedron. Between Earth and Venus the icosahedron, and between Venus and Mercury the octahedron. The fits were not exact, and Mercury's orbital sphere touched the edges rather than faces of the octahedron. But the young astronomer was convinced, and at the age of just 25, he had not only explained the 'cosmic Mystery', but had discovered the underlying principle of the divine harmony inherent in the universe.

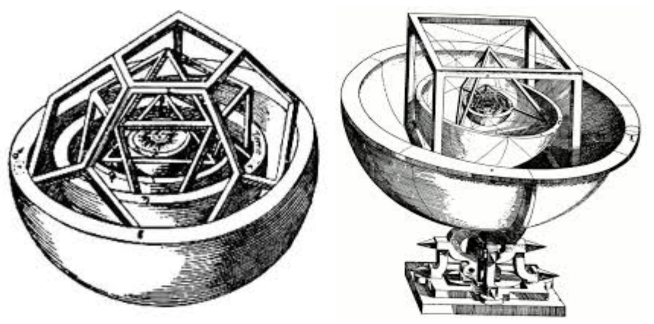


Figure 6 Kepler's illustration of how the Platonic solids nest into the planetary spheres. From *Mysterium Cosmographicum*.

But there is more to the *Mysterium Cosmographicum*. In the second part Kepler checks the proportions of his model against the observed data. He admits that his calculations demonstrate just how rough and ready the model is. The planets move in eccentric orbits around the Sun, so that their distance from the sun varied continuously. Kepler gave his planetary spheres sufficient thickness to accommodate this variation, with inner and outer walls representing minimum and maximum distance from the sun (perigee and apogee). He blamed other discrepancies on the unreliability of Copernicus' data, but he went along with Copernicus by accepting that the spheres, as well as the Platonic solids, were geometric constructs, rather than physical entities. Much of the second part of *Mysterium Cosmographicum* deals with defending a model he already knew to be inexact.

Kepler's Astonishing Idea

But in the final section of *Mysterium Cosmographicum*, by attempting to answer a simple question which no one had asked before, Kepler demonstrated his genius.

Having convinced himself that there can only be six planets in our solar system (because there are only five perfect Platonic solids to fit between them), and that their distances from the sun are what they are because of the geometry of the Platonic solids, Kepler moves on to inquire into the relationship between a planet's distance from the sun and the length of its year, the time needed for a complete revolution around the sun (its period). He discovered, to his surprise, that the further a planet is away from the sun the slower it travels, i.e. it not only does it have a greater distance to travel along its orbit, but it does so at a slower pace. This was of course all part of God's plan, but Kepler wanted to know why.

He reasoned that there must be a force emanating from the sun which drives the planets along their orbits. The outer planets move more slowly because this driving force decreases as the distance increases, 'as does the force of light'. In the second edition of *Mysterium Cosmographicum* (published in 1621), he explained himself: 'There was once a time when I firmly believed that the motive force of a planet was a soul. . . Yet, as I reflected that this cause of motion diminishes in proportion to its distance from the sun, I came to the conclusion that this force must be something substantial.' By 'substantial' he meant 'an unsubstantial entity emanating from a substantial body', in the same way that for Kepler light was something 'substantial' because it emanated from a substantial sun [15].

Kepler went further. If the force which moves the planets emanates from the sun, why was the sun not the centre of their orbits; in other words, why were the orbits eccentric? He reasoned that there must be a second force located in the planet itself, which opposed the force of the sun. The 'unsubstantial entity emanating from the sun', and the even more mysterious force opposing it, were the first intimations of gravity and inertia in the history of human consciousness [16].

Throughout these deliberations Kepler is attempting to validate his theory that the relationship between a planet's distance from the sun and its orbital period is a linear one, even though he senses that it will turn out wrong. He takes his readers into his confidence, and confesses that his calculations do not support his theory. He laments: 'Though I could have foreseen this from the beginning, I nevertheless did not want to withhold from the reader this spur to further efforts. Oh, that we could live to see the day when both sets of figures agree with each other' [17].

Kepler did live to see that day. In the second edition of *Mysterium Cosmographicum* (1621) he added to the paragraph quoted above: 'We have lived to see this day after twenty-two years and rejoiced in it.' He had figured out the answer two years earlier, in the final section of *Harmonice Mundi*. He presented it to the world as the eighth of a list of thirteen propositions which he needed for his investigation of celestial harmonies. For posterity it became known as Kepler's third law of planetary motion, arguably the crowning achievement of his life's work, although Kepler did not see it this way. It says a great deal about the working of Kepler's mind that he considered his third law, as well as the first two, as mere by-products of his quest to prove beyond doubt the fundamental harmony in God's creation.

Interlude: *Astronomia Nova* and the first two Laws of Planetary Motion

In 1600 Kepler moved to Prague where he began to analyse Tycho Brahe's closely guarded astronomical data. Upon the latter's death in 1601 Kepler was appointed imperial mathematician to the emperor Rudolf II, and gained full access to Brahe's observations, which he used to determine the orbit of Mars. A task which he thought he would manage in a couple of weeks, took him four years, in large part owing to his refusal to relinquish a circular orbit. He was able to establish the second law (on their journey round the sun planets sweep out equal areas in equal times) in less than a year by continuing to assume an eccentric circular orbit. But it took another three years before the accuracy of Brahe's observations forced him to conclude that the orbit of Mars was an ellipse (today referred to as his first law, which applies to all the planets).

Kepler liked to confide in his readers. Here is a passage from *Astronomia Nova* describing his mental agonies over the elliptical orbits: 'Why should I mince my words? The truth of Nature, which I had rejected and chased away, returned by stealth through the back door, disguising itself to be accepted. . . I thought and searched, until I went nearly mad, for a reason why the planet preferred an elliptical orbit (to my circular one). . . . Oh, what a foolish bird I have been!' [18].

Although he had largely completed the task by 1605, publication was delayed for four years by legal battles with the heirs of Tycho Brahe, who were the rightful owners of Tycho's observations, but which Kepler had purloined shortly after his death, as he admitted in 1605, when he wrote to his friend and fellow astronomer Christopher Heydon:

'I confess that when Tycho died I quickly took advantage of the absence, or lack of circumspection of the heirs, by taking the observations under my care, or perhaps usurping them . . .' [19].

A settlement was finally made in 1609, with Kepler agreeing to an introduction written by Franz Tengnagel, Brahe's son-in-law and one-time assistant. *Astronomia Nova* (*A new Astronomy*) describes Kepler's tortuous journey towards establishing the orbit of Mars, during the course of which he took his readers round every wrong turn and blind alley. Even so, he was lucky. The eccentricity of Mars is second only to that of Mercury (which being so close to the sun is difficult to observe with any degree of accuracy). Jupiter and Saturn have eccentricities half of that of Mars, and that of Venus is even less. (See table 3 below for the eccentricities of the planets.) It is unlikely that Kepler would have been able to calculate any of the other orbits using Brahe's observations.

He once likened [20] the ellipses to a cartload of dung, which he was forced against his will to bring into his first law, in order to get rid of a much larger cartload of dung, namely, Ptolemy's cycles, epicycles, and equants. Yet later, in *Harmonice Mundi*, he realised that if the orbits of the planets really were circles, there would be no celestial harmony! The elliptical orbits were destined to reveal a music of the spheres more subtle than any that had gone before.

Years of Sorrow in Linz

In 1611 Kepler's wife Barbara began having epileptic seizures, and died soon afterwards. In the same year his six year old son Friedrich died of smallpox. Religious and political tensions in Prague forced Kepler to relocate to Linz in 1612. A teaching post at a small district school was set up for or him by his supporters when it became clear that after the death of the emperor Rudolf II in 1612, he could no longer stay in Prague. The regular salary augmented the rather haphazard income from

Matthias II who had succeeded his brother Rudolf II as emperor, and had reaffirmed Kepler's position as imperial mathematician.

Kepler was as open and honest about his beliefs as he was about his calculations, and although his protestant beliefs were tolerated, he was forbidden to take part in the Lutheran Divine Service in Linz because he refused to sign up to the dogma of the omnipresence (*Ubiquitas*) of Christ's body and blood in the bread and wine of the eucharist, as promulgated in the 1577 Formula of Concord, the authoritative Lutheran statement of faith. He was however allowed by special dispensation to participate in the celebration of the eucharist in a nearby village with a more open-minded minister.

In 1615 he received word from his sister that his mother, Katharina, had been accused of witchcraft. If the accused did not confess, it was common to use torture to extract a confession; and regardless of whether the victim confessed, a cruel and merciless death awaited her. Kepler's fame was able to circumvent the worst, but his mother's fate preyed on his mind for the following six years. The (ultimately successful) attempt to save her required many carefully worded letters, and several visits to the consistorial council in Stuttgart. Katharina was imprisoned for more than a year and subjected to verbal interrogation, which included a graphic description of the torture which would follow unless she confessed. Kepler drew up a detailed legal defence, and was able to show that the evidence against his mother amounted to no more than malicious rumours. She was finally released in 1621.

The death of his wife and subsequent legal conflict over her will, the religious controversies, his mother's witch trial, the battle he fought with his conscience; all these tribulations massed like dark clouds over his soul, a darkness further deepened by the death of two of his children in 1617 and 1618. But none of this diverted him from completing what he considered the crowning glory of his life; publication of *Harmonice Mundi* in 1619. From these seven years of darkness broke forth a grand musical interpretation of the harmony of the world.

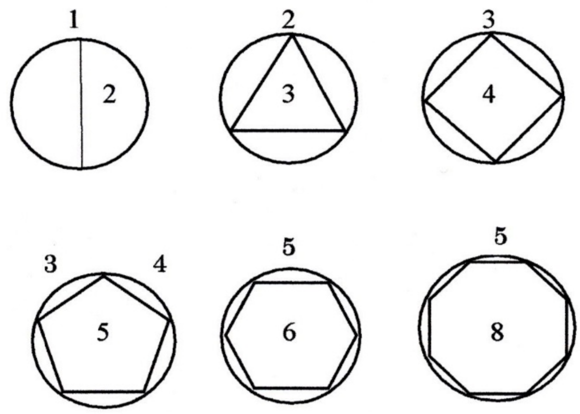
Harmonices Mundi

The full title of Kepler's most important book is '*Ioannis Keppleri Harmonices Mundi Libri V*' (*Johannes Kepler's World Harmony in Five Books*), which was published in the summer of 1619. He dedicated it to King James VI and I of Great Britain, possibly sensing in him a worthy successor to Emperor Rudolf II as a patron of Rosicrucian and hermetic interests. In the first two books Kepler discusses two and three dimensional geometry; in the other three he concerns himself with music, and the closely related subjects of astronomy and astrology.

Kepler picks up more or less where he had left off 22 years earlier in *Mysterium Cosmographicum*. He refined his ideas on geometric harmony, and investigated tessellation, the rules by which one or more different regular polygons tessellate, i.e. cover a plane area with no gaps or overlaps. He extended these ideas to three dimensions in the five regular polyhedra (the Platonic solids), and the thirteen irregular polyhedra (the Archimedean solids) [21]. He also investigated the four regular star polyhedra (the Kepler-Poinsot solids).

Having re-established for himself the harmony inherent in geometry, and remembering the flash of insight he had in July 1595, Kepler now attempted to discover the basis for the seven primal consonant intervals in geometric relationships, rather than in numerical ones: 'I don't want to prove anything by means of mystical numbers, and I also believe that such proofs are not possible.' He therefore tried to explain them (at least to his own satisfaction) by relating them to regular polygons.

He imagined the two ends of a string joined to form the circumference of a circle, which was then divided by regular polygons. As there are infinitely many regular polygons, he limited himself to those which can be constructed using only straightedge and compasses (as the Greeks did). He considered other regular polygons such as the heptagon and nonagon as 'unknowable' [22], and not used by God to 'embellish the world'. The first five constructible regular polygons are the triangle, the



square, the pentagon, the hexagon, and the octagon.

The divisions of the string are listed in such a way that the sum of the numerators and denominators of the resulting fractions provides the denominator of the next fraction. From each fraction two branches arise, but the sequence is broken off as soon as the sum of the numerator and denominator gives rise to a polygon which cannot be constructed with straightedge and compasses only.

See fractions highlighted in yellow in table 1. He did not include the two red highlighted fractions, because when inverted these are the fourth and major third intervals raised an octave, which he had already found an octave lower [23].

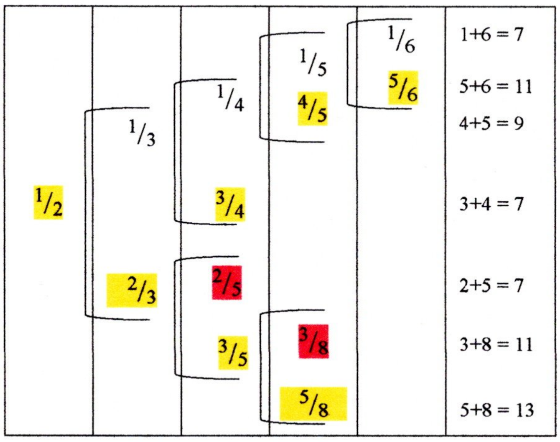


Table 1 Kepler's derivation of the seven 'primal' interval ratios

From these geometrically derived harmonic intervals Kepler derived the remaining intervals in the diatonic scale. He kept the Pythagorean second (9:8), (the *epogdoon*); derived the major seventh (15:8) by 'narrowing' the Pythagorean seventh (243:128) [24], and derived the minor seventh (9:5) by further 'narrowing' the major seventh [25].

Ratio	Interval
2:1	Octave
3:2	Perfect fifth
4:3	Perfect fourth
5:3	Major sixth
5:4	Major third
8:5	Minor sixth
6:5	Minor third

It is possible to illustrate the relationships between these intervals in a circle of consonant harmonic intervals. See Figure 7. The angles are calculated by multiplying $\log_2$ of each interval by 180° [26].

Figure 8 shows the harmonic divisions of a musical string. When inverted, the fractions become the ratios of the consonant intervals which Kepler found using regular polygons.

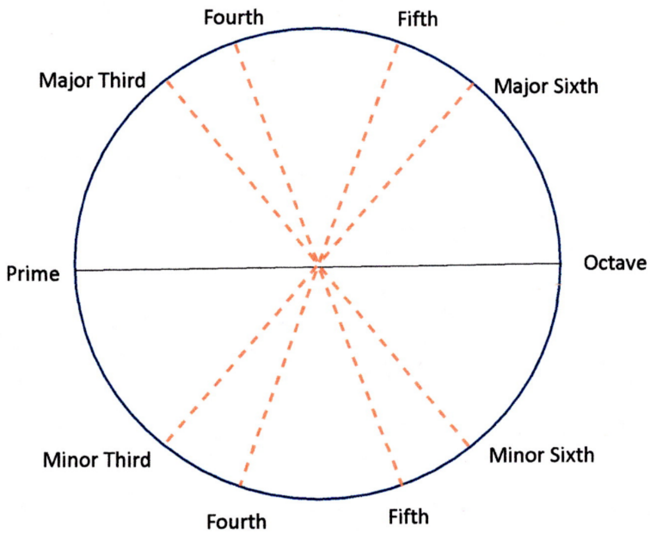


Figure 7 The circle of Kepler's 'primal' harmonic intervals.

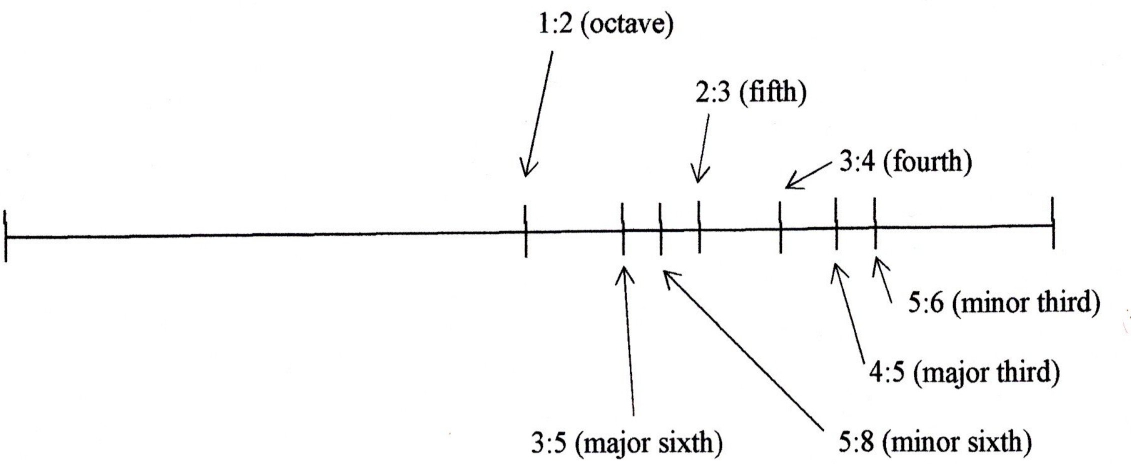


Figure 8 The harmonic divisions of a musical string.

Kepler's next step was to locate these ratios in the movements of the planets. Previous theories of cosmic harmony had assigned a single tone to each planet (because of their assumed circular orbits). Kepler worked long and hard on this question. In his first attempt he showed that no harmonic intervals are to be found in the orbital periods of the planets. He drew a blank with his second attempt which compared the ratios of the distances at perigee and apogee. He failed in his third attempt which tried the ratios of the maximum and minimum orbital speeds of the planets. Next he tried to relate the times it took the planets to cover a unit distance along their orbits. Again, no luck.

Kepler was not one to give up easily, and after considerable trial and error, all described in the usual selfcritical detail, he realised that with elliptical orbits the pitch of each tone would vary in proportion to the planets' varying orbital speeds. At long last he found what he was looking for.

But first he explained for the benefit of his readers (and possibly Aristotle!) how it was possible to perceive sound through the movements of the planets: 'In fact, there are no real sounds in the heavens, and the movement is not so turbulent that a whistling is produced by friction with the heavenly air.' [27] Kepler explained that the celestial harmonies are carried to us by the light with which we see the planetary movements, and it is the light which enables us to 'hear' the music in our minds.

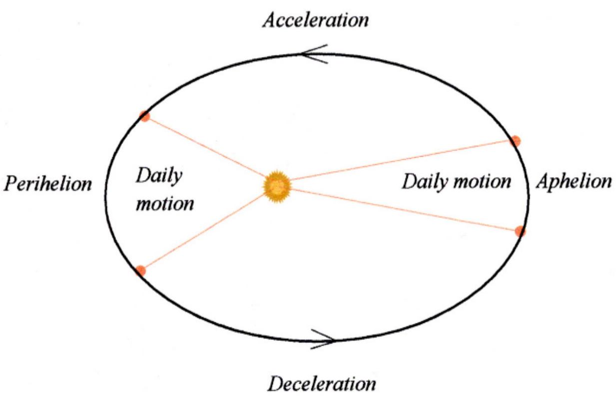


Figure 9 Sketch of an elliptical orbit (eccentricity and daily motion greatly exaggerated).

He imagined that the daily angular motion as seen from the sun was a measure of the frequency of a musical tone. It follows from Kepler's second law that a planet moves faster in perihelion (when closer to the sun), slows down towards aphelion (further away from the sun), then speeds up again. As the velocity changes along the planet's orbital path, so does the tone, running through a musical interval the size of which depends on the eccentricity of the orbit. The actual pitch of the tone depends on the (average) distance between the Sun and the planet. It was in exploring this relationship that Kepler discovered the third law of planetary motion.

Kepler's Third Law

Whereas the first two laws describe the movement of single planets, the third law describes the relationship between their movements. Kepler defines the third law in the third chapter of Book Five. Here he lists 13 'Propositions of Astronomy which are needed for the Investigation of the Celestial Harmonies', which he considered essential for his exposition and proof that musical harmonies are to be found in the movement of the planets. His third law is the eighth 'Proposition', merely one of the premises for his much larger goal.

The third law states that the square of a planet's period of rotation is proportional to the cube of its semi-major axis (the average distance from the Sun during perihelion and aphelion) [28].

The third law (which of course wasn't called that yet) is the only one of the 13 'Propositions' for which Kepler gives the date of its discovery. He did this not because he considered it important for the further development of astronomy (which of course it was), but because this particular 'Proposition' provided the answer to the question he had to leave unanswered in *Mysterium Cosmographicum*, and which had occupied him for the past twenty-five years.

Finding the Music

The first example Kepler gave of how the two numbers of a musical interval are related to the angular orbital speed of a planet was that of Saturn.

The maximum speed of Saturn at perihelion is 133 seconds of arc (arcseconds) per day, its minimum speed at aphelion is 106 arcseconds per day. The ratio 133:106 is very close to 5:4, the ratio of a major third ($5×274×27 =135108$). He concluded that the music of Saturn is contained within the interval of a major third.

Proceeding in this way, he calculated the intervals for every planet, but found that only four of them were consonant (Saturn, Jupiter, mars and Mercury). See Table 3 [29].

Planet	Eccentricity of orbit	Position	Arcseconds per earth day	Dividing the ratio	The intervals Kepler found
Saturn	0.053	Perihelion	133	133:106 = 1.255	Almost 5:4 (1.250) a major third
		Aphelion	106		
Jupiter		Perihelion	330		Almost 6:5 (1.200) a minor

Jupiter	0.048	Perihelion	330	330:270 = 1.222	third
		Aphelion	270		
Mars	0.093	Perihelion	2291	2291:1574 = 1.456	Almost 3:2 (1.500) a perfect fifth
		Aphelion	1574		
Earth	0.017	Perihelion	3678	3678:3423 = 1.074	Almost 16:15 (1.067) a semitone
		Aphelion	3423		
Venus	0.007	Perihelion	5857	5857:5690 = 1.029	Almost 25:24 (1.042) a diësis
		Aphelion	5690		
Mercury	0.206	Perihelion	23040	23040:9840 = 2.341	Almost 12:5 (2.400) an octave plus a minor third.
		Aphelion	9840		

Table 3 Kepler's calculations of the seven planetary harmonic intervals

Mercury's song encompasses more than an octave, while that of Venus hardly changes. Earth's song varies by just a semitone from mi to fa. Kepler couldn't resist commenting that the earth sings 'mi-fa-mi', 'so we can gather even from this that misery and famine reign on our habitat.' [30]

Kepler linked his planetary scales to the medieval modes, but added that they would have a completely different sound. As each planet accelerates and decelerates on its orbital journey, the pitch is constantly changing, and glides from one note to another (glissando), sounding more like a cosmic siren than a musical scale. With Saturn taking 30 years to sing the three notes of its song, and taking into account the limited vocal range of some of the planets, Kepler admitted that celestial harmonies would occur very infrequently [31]. Harmonies between three planets are fairly common, between four only over centuries, and between five only over millennia. He suspected that if a grand alignment of all seven heavenly bodies could be calculated, it would pinpoint the exact moment of creation, but would never re-occur. Because the motions are in irrational proportions to each other, 'they will never return to their starting point, even after infinite ages'.

He widened his search for the remaining consonant intervals by considering what he called the convergent and divergent daily movement of pairs of adjacent planets; that is, he constructed new ratios as follows.

- A convergent ratio is the minimum speed of one planet (at aphelion) : maximum speed of its neighbour (at perihelion),
- A divergent ratio is the maximum speed of one planet (at perihelion) : minimum speed of its neighbour (at aphelion).

Taking the Saturn-Jupiter pair as an example,

Convergent ratio = Jupiter min. speed at aphelion : Saturn max. speed at perihelion

= 270:133 = 2.030, an almost perfect octave.

Divergent ratio = Jupiter max. speed at perihelion : Saturn min. speed at aphelion

= 330:106 = 3.113, close to an octave plus a fifth.

Proceeding in this manner he found the intervals shown in Table 4, [33]. The last column shows the intervals brought into a single octave.

Kepler has found six of his seven primal consonant intervals, and attentive readers will have noticed the obvious omission; there is no sign of the perfect fourth. This might have been acutely embarrassing for him, had he not called upon the one heavenly body he had not yet considered.

Although the moon is not considered a planet in an astronomical sense, ancient traditions have always seen it as one of the seven planets in an esoteric sense [34]. Fortunately for Kepler, it turns out that the ratio constructed from the orbital speeds of the moon as seen from the earth at perigee and apogee is that of a perfect fourth!

Planet Pair	Type of ratio	Dividing the ratio	The intervals Kepler found	Intervals

Saturn - Jupiter	Convergent	270:133 = 2.030	Almost 2:1 (2.000) an octave	Octave
	Divergent	330:106 = 3.113	Not far from 3:1 (3.000) an octave plus a fifth	Fifth
Jupiter - Mars	Convergent	1547:330 = 4.770	Almost 24:5 (4.800) double octave plus a minor third	Minor third
	Divergent	2291:270 = 8.485	Not far from 8:1 (8.000) three octaves	Octave
Mars - Earth	Convergent	3423:2291 = 1.494	Almost 3:2 (1.500) a perfect fifth	Fifth
	Divergent	3678:1547 = 2.378	Almost 12:5 (2.400) an octave plus a minor third	Minor third
Earth - Venus	Convergent	5690:3678 = 1.547	Not far from 8:5 (1.6) a minor sixth	Minor sixth
	Divergent	5857:3423 9= 1.711	Not far from 5:3 (1.667) a major sixth	Major sixth
Venus - Mercury	Convergent	9840:5857 = 1.680	Almost 5:3 (1.667) a major sixth	Major sixth
	Divergent	23040:5690 = 4.049	Almost 4:1 (.000) a double octave	Octave

Table 4 Kepler's additional calculations of the seven planetary harmonic intervals

Kepler would not have been Kepler if he had left it at that. He now needed to discover why God had created the cosmic harmonies. As they cannot be heard on earth, surely some conscious being somewhere, besides God himself, had to be able to hear them. In the epilogue to *Harmonice Mundi*, an essay in praise of the sun, Kepler suggested that the 'intellect' (self-consciousness) best able to appreciate the planetary harmonies might reside in the place where they originated, namely, in the sun.

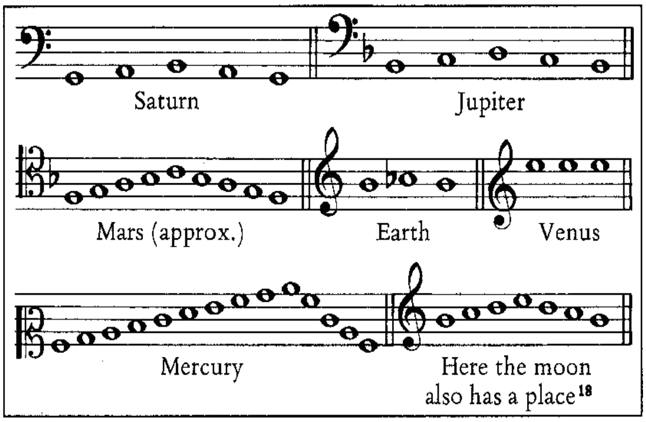


Figure 10 Kepler's planetary scales in modern notation. Saturn's notes sound one octave lower, Mercury's notes one octave higher than written [32]. From https://hermetic.com/godwin/kepler-and-kircher-on-the-harmony-of-the-spheres

'For whose use are all these furnishings, if the globe (the sun) is empty? Indeed, do not the senses themselves cry out that fiery bodies dwell here which are receptive of simple intellects, and that in truth the sun is, if not the king, at least the palace of intellectual fire?'

The Golden Vessels of the Egyptians

In the preamble to the Fifth Book of *Harmonice Mundi* Kepler wrote [35]:

'As regards that which I prophesied two and twenty years ago (in the *Mysterium Cosmographicum*), as regards that of which I was firmly persuaded in my own mind before I had seen Ptolemy's *Harmonics*, . . . for the sake of which I spent the best part of my life in astronomical speculations, visited Tycho Brahe, and took up residence at Prague: finally, as God the Best and Greatest, Who had inspired my mind and aroused my great desire, prolonged my life and strength of mind and furnished the other means through the liberality of the two Emperors and the nobles of this province of Upper Austria, . . I have finally brought to light.'

He goes on to explain that he had been given a copy of Ptolemy's '*Harmonics*', by his mentor Herwart von Hohenburg, as he was composing the first four books of *Harmonice Mundi*. Although Ptolemy's work gave him 'an extraordinary augmentation of my desire and incentive for the job', he felt that it was outdated, that Ptolemy's astronomy 'was far from being of age', and that the 'crudeness of the ancient philosophy' compared poorly with the 'exact agreement in our meditations'. He considered it divine providence, 'the finger of God', that two men centuries apart shared 'the same conception as to the configuration of the world, although neither had been the other's guide in taking this route.'

Kepler continues, 'But now since the first light of dawn eight months ago, since the light of day three months ago, and since the sun of my wonderful speculation has shone fully a very few days ago: nothing holds me back. I am free to give myself up to the sacred madness, I am free to taunt mortals with the frank confession that I am stealing the golden vessels of the Egyptians, in order to build of them a temple for my God, far from the territory of Egypt. If you pardon me, I shall rejoice; if you are angry, I shall bear up. The die is cast, and I am writing the book - whether to be read by my contemporaries or by posterity matters not. Let it await its reader for a hundred years, if God Himself has been waiting six thousand years for a witness.'

Kepler's allusion to the golden vessels of the Egyptians is sometimes interpreted as a riposte towards Ptolemy's '*Harmonics*', in that he (Kepler) had achieved what Ptolemy had not. But Kepler had studied the Old Testament as a student, and he is referring here not to Ptolemy, but to a story told in Exodus. The Hebrews, still captive in Egypt, purloined gold and silver 'jewels' and 'ornaments' from their Egyptian neighbours by 'borrowing' them, and then decamping with them across the Red Sea. When the time came for them to build a tabernacle in the desert, they 'offered' them to their Lord [36].

An interesting rider to this story is that only in Martin Luther's translation of the Old Testament are the stolen objects referred to as 'vessels' (*Gefäße*). More recent German translations, such as that of the Hebrew scholar Martin Buber, describe the stolen objects as 'utensils' (*Geräte*), which is indeed closer to the Hebrew original, so perhaps Luther was thinking of cooking vessels.

The Egyptians were unlikely to lend out their golden utensils, let alone their jewels, to mere slaves, so this story (like so much in the Old Testament) has a deeper meaning. It seems likely that Kepler imagined the Hebrew people taking with them into the Promised Land Egyptian wisdom concerning the ancient teachings of the harmonic structure of the universe, wisdom that through sheer hard work he himself had been able to re-discover, and offer to future generations.

This fits in with Rudolf Steiner's assertion that Kepler's lifelong quest for the harmony of the universe can be traced back to an ancient Egyptian incarnation when he had been a pupil of the Egyptian priests [37]. In spite of a difficult childhood, ill health, religious persecution, war, the deaths of several family members, in short, a life fraught with difficulties and hardship, the harmony he had experienced in an earlier life was his certain guide on an enterprise unique in the history of human consciousness [38].

'*Behold, I have now completed the work which has been my vocation, having employed all the power of my mind which you gave me; I have revealed the glory of your works to those who will read my presentation, as much of their infinite riches as the narrows of my intellect could conceive*.'

Kepler's prayer of thanks at the end of Book Five of *Harmonice Mundi.*

Coda

Stanley Kubrick's iconic science fiction film *2001 A Space Odyssey* was released in 1968, the same year the Moody Blues went in search of the lost chord. With the first moon landing imminent, interest in space was widespread, and 'spaced out' music was popular (especially among the Woodstock generation). In Kubrick's film the opening bars of Richard Strauss' *Also Sprach Zarathustra* (sunrise), and György Ligeti's pieces *Lux Aeterna* and *Atmosphères* introduced many people to music they might otherwise never have heard, but of course it bore no relation to the cosmic harmony discovered, and possibly even 'heard' mentally, by Kepler.

During the course of the 20th Century a number of composers of 'classical' music felt inspired by cosmic space, but few, if any, appear to have been inspired by Kepler's cosmic harmonies, or used his planetary scales, in their compositions. The best known composition to pay tribute to the planets themselves is *The Planets*, composed in 1916 by Gustav Holst. But Kepler's planetary songs are not made manifest, and the musical qualities of the planets are based almost entirely on the characteristics of the Roman gods after whom they are named.

Paul Hindemith (1895-1963) wrote an opera *Die Harmonie der Welt* (*The Harmony of the World*), for which he wrote his own libretto. It was first performed in 1957 in Munich. It was inspired more by Kepler's anxieties and the complexities of his life than by the heavenly harmonies he had discovered. A symphonic suite has been drawn from the opera.

In 1973 the Polish composer Henryk Gorecki was commissioned to write a symphony to celebrate the 500th anniversary of the birth of his compatriot Copernicus. It became his second symphony (the Copernican), and includes text from *De revolutionibus orbium coelestium*.

Philip Glass wrote an opera *Cosmic Symphony to Johannes Kepler*, which premiered in Linz in 2009. It is less concerned with Kepler the man, than with his obsessive search for cosmic harmony, and his hope that he would find answers in the new science of astronomy, the principles of which he was instrumental in establishing [39]. Listeners are expected to decide for themselves how Glass' music corresponds to Kepler's lifelong quest, and how he might have 'heard' it in his mind.

The summer of 2019 commemorates the 400th anniversary of the publication of *The Harmony of the World*. Is there any composer out there composing a tribute to the music Kepler discovered?

Endnotes and References

1. Hans Georg Herwart von Hohenburg (1553–1622) was a Bavarian statesman and scholar, and a patron and correspondent of Kepler.
2. A ratio expresses a relationship between two quantities.
3. Interestingly, from the point of view of this article, Philolaus is also credited with being the first to suggest that the Earth is not the centre of the universe.
4. An overtone is a tone above the fundamental (open string) tone, and can be heard together with it.
5. Raphael lived from 1483 to 1520, and the 'The School of Athens' was painted between 1509 and 1511.
6. The omega (ω) is apparently a Renaissance "typo", and should be an omicron (o).
7. *Epogdoon* means literally 'a number equal to another plus an eighth thereof'. 9/8 = 1 and 1/8.
8. The *Timaeus* was the only one of Plato's dialogues available to medieval and early Renaissance scholars.
9. In order of increasing distance from Earth located at the centre: Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn.
10. '*The final cause, then, produces motion by being loved, but all other things move by being moved.*' Aristotle, *Metaphysics*.
11. https://en.wikipedia.org/wiki/Musica_universalis
12. He used a complex system of epicycles, eccentric deferents, and equants.
13. But the title implies much more; the Latin suffix 'graphicum' implies an 'exquisite', or a 'perfect' cosmos; the Latin 'mysterium' is closely linked to the Greek 'musterion' (μυστεριον), referring both to the sacred mysteries and to the sacraments of Christianity.
14. A polygon is a two-dimensional figure whose sides are all equal in length, so that when drawn inside a circle (Plato's perfect shape) all its corners touch the circumference of the circle, and when a circle is drawn inside a regular polygon, it touches all the sides exactly in their centres.
15. Quoted in Arthur Koestler (1961) *The Watershed, a Biography of Johannes Kepler*, Heinemann's Science Study Series, p 57.
16. More than 150 years later Newton struggled with the same problem. In a letter to Richard Bentley he wrote: '*Tis unconceivable that inanimate brute matter should (without the mediation of something else which is not material) operate upon & affect other matter without mutual contact; as it must if gravitation in the sense of Epicurus be essential & inherent in it. And this is one reason why I desired you would not ascribe (innate) gravity to me. That gravity should be innate, inherent & (essential) to matter so that one body may act upon another at a distance through a vacuum without the mediation of any thing else by & through which their action or force (may) be conveyed from one to another is to me so great an absurdity that I believe no man who has in philosophical matters any competent faculty of thinking can ever fall into it. Gravity must be caused by an agent (acting) constantly according to certain laws, but whether this agent be material or immaterial is a question I have left to the consideration of my readers.*' Source: http://www.newtonproject.ox.ac.uk/view/texts/normalized/THEM00258
17. Quoted in Koestler, p 58.
18. Quoted in Koestler, p 147.
19. Quoted in Koestler. p 161. Heydon (15??-1623) was an English astronomer whose observations of Mars led him to conclude, along with Tycho Brahe, that the hitherto published positions were incorrect.
20. In a letter to the Danish astronomer Longomontanus, whom he met in Prague.
21. The most well-known Archimedean solid is the truncated icosahedron, a.k.a. the soccer ball, with its 20 white regular hexagons and 12 black pentagons.
22. Latin '*inscibilis*'; '*unwissbar*' is the German word Kepler used.
23. $43\times21 = 83$ and $54\times21 = 104 = 52$
24. By use of the syntonic comma K = 80/81 ($243128\times8081 = 158$).
25. By use of the deisis D = 24/25 ($158\times2425 = 95$).
26. Logarithm with base 2.
27. Quoted in Bruno Gingras (2003) Johannes Kepler's *Harmonices mundi*: A "Scientific" Version of the Harmony of the Spheres, Part II, http://adsabs.harvard.edu/full/2003JRASC..97..259G
28. In mathematical shorthand, $T^2 \quad D^3$ or $T^2 = kD^3$ or $T = kD^{1.5}$
29. Adapted from Bruno Gingras, op.cit.
30. Quoted in Koestler, p213.
31. Bruno Gingras, op.cit.
32. The not often used C clef shows the Martian scale to run from F to C, and Mercury's from A to C in the next octave.

33. Adapted from Bruno Gingras, op.cit.
34. As, for example, in the days of the week.
35. http://www.24grammata.com/wp-content/uploads/2014/08/Kepler-Harmonies-Of-The-World-24grammata.pdf
36. See Exodus 3;22, 11;2, 12;35-36, and 25;1-9.
37. In a lecture held in Leipzig on 14 September 1908 in *Egyptian Myths and Mysteries* (GA106), and one in Hamburg on 25 May1910, in *Manifestations of Karma* (GA120).
38. In 1629, the last year of his life, in a letter to his future son in law Jakob Bartsch Kepler wrote: *'When the storm rages, and the state is threatened by shipwreck, we can do nothing more noble than to lower the anchor of our peaceful studies into the ground of eternity'.*
39. https://philipglass.com/films/kepler/

Autumn 2019 *Maarten Ekama*

Events

Annual Meeting of the Science and Maths Group (of the AS in GB)

Saturday 31st October 10 am - 4 pm
Field Centre Nailsworth GL6 0QE

If you intend to come ***please get in touch*** as we will need to arrange the day according to the numbers and may have to cancel at the last minute if regulations change.
If you have any initiatives to share but are unable to attend please also get in touch.
simon.charter@live.co.uk or phone 07814 786682

Programme

10 am welcome apologies etc
10.05 *The Beave*r - Judyth Sassoon
11.15 break
11-45 *Boundary phenomena and micro flows*- Philip Kilner
13.00 lunch
1400 *AGM business*, finances , appointment of officers, website, future activities, any other business
14.45 break
15.00 *Jupiter Saturn conjunction*- Alex Murrell
16.00 finish

Colloquium in memory of Jochen Bockemühl

From Saturday, March 13, 2021, 3 pm to Sunday, March 14, 12.30 pm, the Natural Science Section in Dornach will hold a colloquium in memory of Jochen Bockemühl (1928-2020). For information see: https://dasgoetheanum.com/jochen-bockemuehl-1928-2020/..

Grants

Science and Mathematics Group Funding: Call for Applications

We are pleased to announce that small grants are available to members of the Science and Mathematics Group. We can contribute to projects and travel costs (e.g. to conferences). Please contact the treasurer Simon Charter, with a brief proposal outline and a breakdown of costs.
simon.charter@live.co.uk, 01453 882114.

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Note from the Treasurer and Membership Secretary.

The subscription for membership of the Science Group (including receipt of Newsletter) stands at £10 per year. If you have not already done so, please update your standing orders and let me know when this is done. I can still accept cheques but the local bank has closed so paying cheques in is more difficult. Standing orders or direct payment are preferable.

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Next Issue

This newsletter is usually issued to members twice each year, in the spring and autumn. This year

publication of the spring issue was delayed by the covid crisis. The next newsletter is scheduled for spring 2021. Please send copy to the Editor:
js7892@bristol.ac.uk

Disclaimer

The opinions expressed in the published reports and articles are the authors' own and do not necessarily reflect the views of the Editor or members of the Science and Mathematics Group of the AS of GB.

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